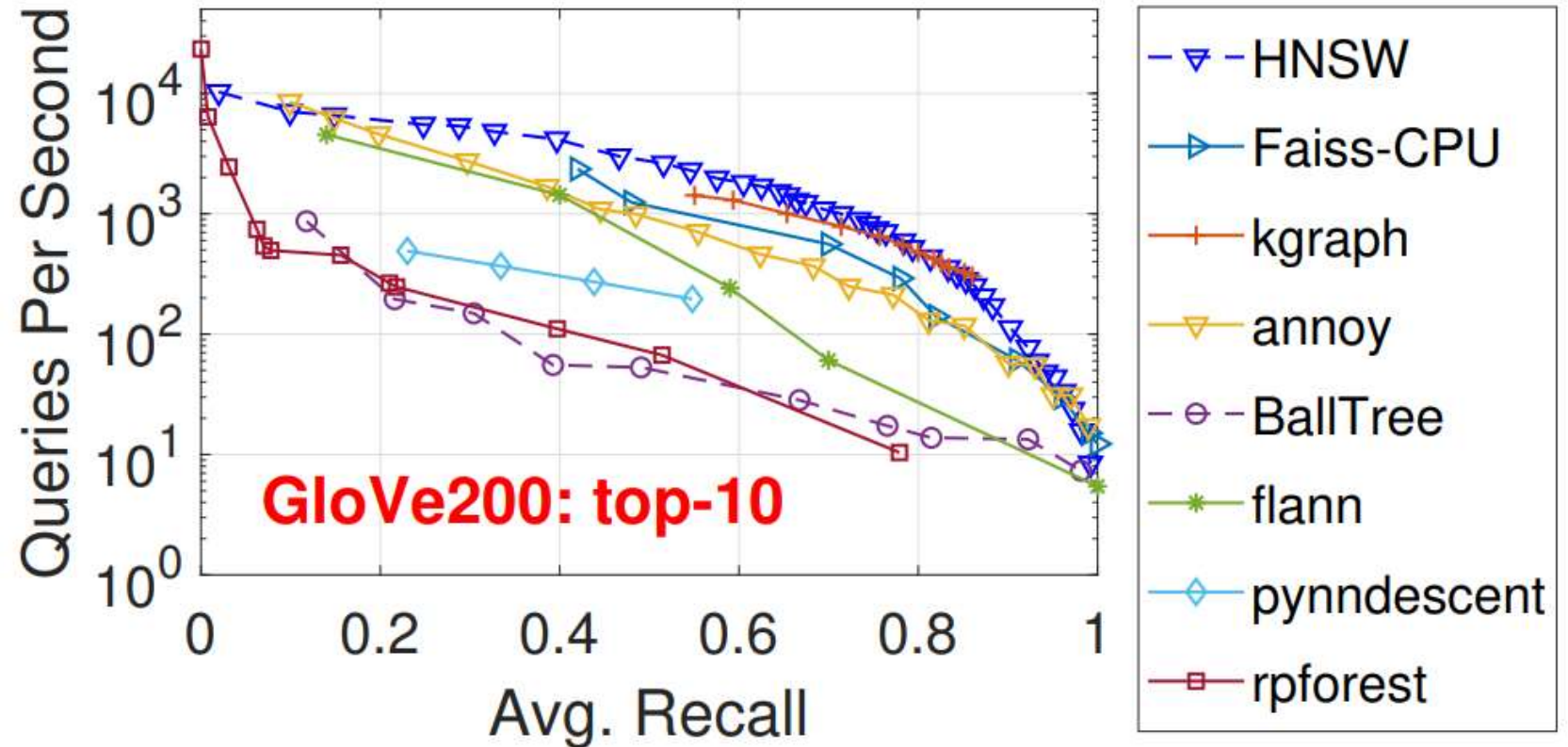


Approximate Nearest Neighbor Search

Weijie Zhao

Approximate Nearest Neighbor Search

- Tree
- Hashing
- Quantization
- Graph

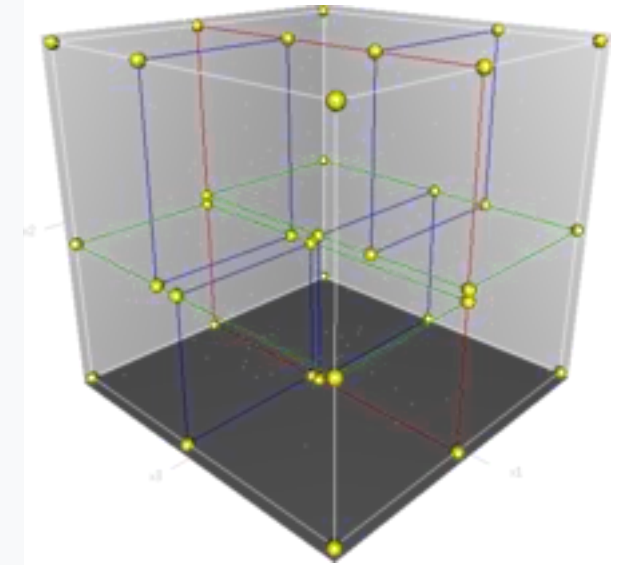


K-D Tree

```
function kdtree (List of points pointList, int depth)
{
    // Select axis based on depth so that axis cycles through all valid values
    var int axis := depth mod k;

    // Sort point list and choose median as pivot element
    select median by axis from pointList;

    // Create node and construct subtree
    node.location := median;
    node.leftChild := kdtree(points in pointList before median, depth+1);
    node.rightChild := kdtree(points in pointList after median, depth+1);
    return node;
}
```



Locality-Sensitive Hashing (LSH)

- Random Projection
- MinHash
- Consistent Weighted Sampling

Algorithm 1 Generalized consistent weighted sampling (GCWS) for hashing the pGMM kernel.

Input: Data vector u_i ($i = 1$ to D)

Generate vector \tilde{u} in $2D$ -dim by (1).

For i from 1 to $2D$

$r_i \sim \text{Gamma}(2, 1)$, $c_i \sim \text{Gamma}(2, 1)$, $\beta_i \sim \text{Uniform}(0, 1)$

$t_i \leftarrow \lfloor p \frac{\log \tilde{u}_i}{r_i} + \beta_i \rfloor$, $a_i \leftarrow \log(c_i) - r_i(t_i + 1 - \beta_i)$

End For

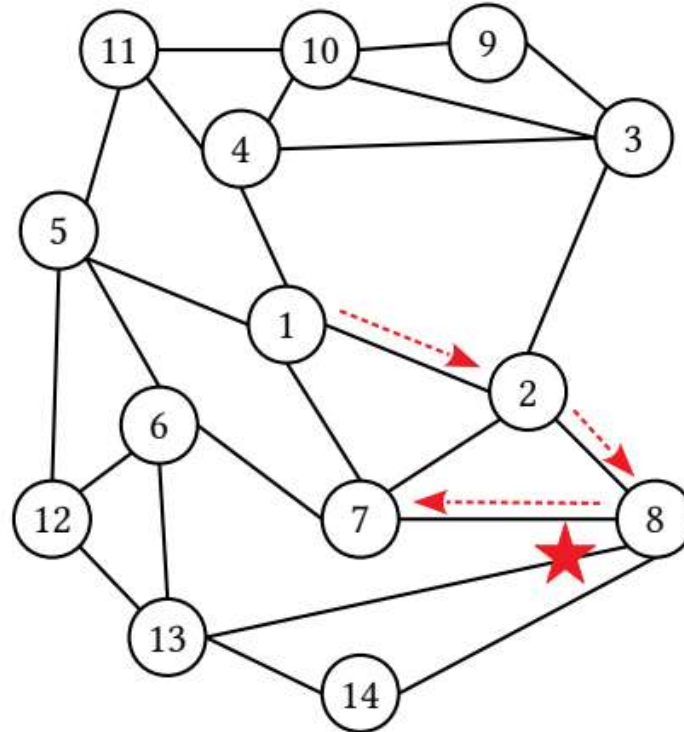
Output: $i^* \leftarrow \arg \min_i a_i$, $t^* \leftarrow t_{i^*}$

Quantization

- Map high dimensional vector to low dimensional integer
- K means
- Product Quantization (PQ)

Graph-Based ANN

- Hierarchical Small World Graph (HNSW)
- Search ON Graph (SONG)
- Bipartite Graph Indices (BEGIN)



Initialization	$q: 1$ $topk: \emptyset$ $visited: 1$
Iteration 1	$q: 2\ 7\ 4\ 5$ $topk: 1$ $visited: 1\ 2\ 4\ 5\ 7$
Iteration 2	$q: 8\ 7\ 3\ 4\ 5$ $topk: 1\ 2$ $visited: 1\ 2\ 3\ 4\ 5\ 7\ 8$
Iteration 3	$q: 7\ 3\ 4\ 5\ 14\ 13$ $topk: 1\ 2\ 8$ $visited: 1\ 2\ 3\ 4\ 5\ 7\ 8\ 13\ 14$
Iteration 4	$q: 3\ 4\ 5\ 6\ 14\ 13$ $topk: 7\ 2\ 8$ $visited: 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 13\ 14$