Tensor Computing in Deep Learning

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02/20/2025

- Scalar
- Vector
- Matrix
- Tensor
  - Rank
  - Dimension

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- Matrix multiplication
- Non-linear activation
- Gradient descent

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- ~~Gradient descent~~

Graduate student descent

Tensor Operations

- Element-wise add
- Element-wise plus
- Element-wise division
- Hadamard product
- Matrix multiplication
- Batched matrix multiplication
- More linear algebra operations…
- Collect, Scatter, Reduce…

Libraries

- Numpy
- Blas
- cuBlas
- cuSparse
- MKL
- TensorFlow
- PyTorch
- MXNet
- …

Lazy Evaluation and Code Generation

```
c = a + b
d = c * 2

for i = 1 to n do
        c[i] = a[i] + b[i]
for i = 1 to n do
        d[i] = c[i] * 2

for i = 1 to n do
        d[i] = (a[i] + b[i]) * 2
```

Graph Optimizer

- Graph minimization and canonicalization
  - Constant Folding
  - Common subexpression elimination
  - Remove unnecessary operations
- Algebraic simplification and reassociation
- Copy propagation

Meta Optimizer

```
i = 0
while i < config.meta_optimizer_iterations (default=2):
    Pruning()             # Remove nodes not in fanin of outputs, unused functions
    Function()            # Function specialization & inlining, symbolic gradient inlining
    DebugStripper()*      # Remove assert, print, check_numerics
    ConstFold()           # Constant folding and materialization
    Shape()               # Symbolic shape arithmetic
    Remapper()            # Op fusion
    Arithmetic()          # Node deduping (CSE) & arithmetic simplification
    if i==0: Layout()     # Layout optimization for GPU
    if i==0: Memory()     # Swap-out/Swap-in, Recompute*, split large nodes
    Loop()                # Loop Invariant Node Motion*, Stack Push & Dead Node Elimination
    Dependency()          # Prune/optimize control edges, NoOp/Identity node pruning
    Custom()              # Run registered custom optimizers (e.g. TensorRT)
    i += 1
```

https://web.stanford.edu/class/cs245/slides/TFGraphOptimizationsStanford.pdf

Constant Folding Optimizer

```
do:
    InferShapesStatically() # Fixed-point iteration with symbolic shapes
    graph_changed = MaterializeConstants() # grad broadcast, reduction dims
    q = NodesWithKnownInputs()
    while not q.empty():
      node = q.pop()
      graph_changed |= FoldGraph(node, &q) # Evaluate node on host
    graph_changed |= SimplifyGraph()
while graph_changed
```

Constant Folding Optimizer: SimplifyGraph()

- Removes trivial ops, e.g. identity Reshape, Transpose of 1-d tensors, Slice(x) = x, etc.
- Rewrites that enable further constant folding
- Arithmetic rewrites that rely on known shapes or inputs, e.g.
  - Constant push-down:
    - Add(c1, Add(x, c2)) => Add(x, c1 + c2)
    - ConvND(c1 * x, c2) => ConvND(x, c1 * c2)
  - Partial constfold:
    - AddN(c1, x, c2, y) => AddN(c1 + c2, x, y),
    - Concat([x, c1, c2, y]) = Concat([x, Concat([c1, c2]), y)
  - Operations with neutral & absorbing elements:
    - x * Ones(s) => Identity(x), if shape(x) == output_shape
    - x * Ones(s) => BroadcastTo(x, Shape(s)), if shape(s) == output_shape
    - Same for x + Zeros(s) , x / Ones(s), x * Zeros(s) etc.
    - Zeros(s) - y => Neg(y), if shape(y) == output_shape
    - Ones(s) / y => Recip(y) if shape(y) == output_shape

Arithmetic Optimizer

```
DedupComputations():
    do:
      stop = true
      UniqueNodes reps
      for node in graph.nodes():
         rep = reps.FindOrInsert(node, IsCommutative(node))
         if rep == node or !SafeToDedup(node, rep):
           continue
         for fanout in node.fanout():
            ReplaceInputs(fanout, node, rep)
         stop = false
    while !stop
```

Arithmetic Optimizer

- Arithmetic simplifications
  - Flattening: a+b+c+d => AddN(a, b, c, d)
  - Hoisting: AddN(x * a, b * x, x * c) => x * AddN(a+b+c)
  - Simplification to reduce number of nodes:
    - Numeric: x+x+x => 3*x
    - Logic: !(x > y) => x <= y
- Broadcast minimization
  - Example: (matrix1 + scalar1) + (matrix2 + scalar2) => (matrix1 + matrix2) + (scalar1 + scalar2)
- Better use of intrinsics
  - Matmul(Transpose(x), y) => Matmul(x, y, transpose_x=True)
- Remove redundant ops or op pairs
  - Transpose(Transpose(x, perm), inverse_perm)
  - BitCast(BitCast(x, dtype1), dtype2) => BitCast(x, dtype2)
  - Pairs of elementwise involutions f(f(x)) => x (Neg, Conj, Reciprocal, LogicalNot)
  - Repeated Idempotent ops f(f(x)) => f(x) (DeepCopy, Identity, CheckNumerics...)
- Hoist chains of unary ops at Concat/Split/SplitV
  - Concat([Exp(Cos(x)), Exp(Cos(y)), Exp(Cos(z))]) => Exp(Cos(Concat([x, y, z])))
  - [Exp(Cos(y)) for y in Split(x)] => Split(Exp(Cos(x), num_splits)

Differentiation

- Numerical differentiation
- Symbolic differentiation
  - Chain rules
- Forward mode auto differentiation
- Reverse mode auto differentiation