Small Ramsey Numbers

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first version: June 11, 1994; revision #17: June 7, 2024
http://www.combinatorics.org

ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, multicolor and hypergraph Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but rather we concentrate on their specific values.

Mathematical Reviews Subject Number 05C55

Revisions

1993, February	preliminary version, RIT-TR-93-009 [Ra2]
1994, July 3	posted on the web at the *ElJC*
1994, November 7	*ElJC* revision #1
1995, August 28	*ElJC* revision #2
1996, March 25	*ElJC* revision #3
1997, July 11	*ElJC* revision #4
1998, July 9	*ElJC* revision #5
1999, July 5	*ElJC* revision #6
2000, July 25	*ElJC* revision #7
2001, July 12	*ElJC* revision #8
2002, July 15	*ElJC* revision #9
2004, July 4	*ElJC* revision #10
2006, August 1	*ElJC* revision #11
2009, August 4	*ElJC* revision #12
2011, August 22	*ElJC* revision #13
2014, January 12	*ElJC* revision #14
2017, March 3	*ElJC* revision #15
2021, January 15	*ElJC* revision #16
2024, June 7	*ElJC* revision #17

1. Scope and Notation

There is vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey papers [GrRö, Neš, ChGra2, Ros2]), but the progress on evaluating the basic numbers themselves has been unsatisfactory for a long time. In the last few decades, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments. If $s = 2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we may write k instead of G_i, and if $G_i = G$ for all i we may use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s = 2$, $K_k - e$ denotes a K_k without one edge, and for $s = 3$, $K_k - t$ denotes a K_k without one triangle (hyperedge).

The graph nG is formed by n disjoint copies of G, $G \cup H$ stands for vertex disjoint union of graphs, and the **join** $G + H$ is obtained by adding all of the edges between vertices of G and H to $G \cup H$. P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of the cycle C_{i-1} (thus $W_i = K_1 + C_{i-1}$). $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \overline{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. The **fan** graph F_n is defined by $F_n = K_1 + nK_2$. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively, and $\delta(G)$ and $\Delta(G)$ minimum and maximum degree of G. Finally, $\chi(G)$ denotes the chromatic number of G. In general, we follow the notation used by West [West].

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, Section 3 for the much studied two color cases of $K_n - e$, K_3, $K_{m,n}$, and Section 4 for numbers involving cycles. Section 5 lists other often studied two color cases for general graphs. The multicolor and hypergraph cases are gathered in Sections 6 and 7, respectively. Finally, Section 8 gives pointers to cumulative data and to other surveys.

2. Classical Two-Color Ramsey Numbers

2.1. Values and bounds for $R(k, l)$, $k \leq 10$, $l \leq 15$

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 41	47 50	53 59	60 68	67 77	74 87
4		18	25	36 41	49 61	59 84	73 115	92 149	102 191	128 238	138 291	147 349	158 417
5			43 48	59 87	80 143	101 216	133 316	149 442	183 633	203 848	233 1138	267 1461	275 1878
6				102 165	115 298	134 495	183 780	204 1171	262 1804	294 2566	347 3703	 5033	401 6911
7					205 540	219 1031	252 1713	292 2826	405 4553	417 6954	511 10578	 15263	 22112
8						282 1870	329 3583	343 6090	457 10630	 16944	817 27485	 41525	873 63609
9							565 6588	581 12677	 22325	 38832	 64864		
10								798 23556	 45881	 81123			1313

Table Ia. Known nontrivial values, lower bounds (2024) and upper bounds (2017) for two-color Ramsey numbers $R(k, l) = R(k, l; 2)$, for $k \leq 10$ and $k \leq l \leq 15$. For the best known upper bounds (2024) with $k \geq 4$ see Table Ib.

k \ l	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	Kéry	Ka2 GrY	GR McZ	Ka2 GR	Ex5 Ang	Ex20 GoeR1	Kol1 Les	Kol1 GoeR1	Kol2 GoeR1	Kol2 GoeR1
4	GG	Ka1 MR4	Ex19 MR5	Ex3 Mac	ExT Mac	Ex16 Mac	HaKr1 Mac	ExT Spe4	SuLL Spe4	ExT Spe4	ExT Spe4	Tat Spe4
5		Ex4 AnM1	Ex25 HZ1	CaET HZ1	HaKr1 Spe4	Kuz Mac	ExT Mac	Kuz HW+	Kuz HW+	Kuz HW+	Kuz HW+	2.3.h HW+
6			Ka2 Mac	ExT HZ1	ExT Mac	Kuz Mac	Kuz Mac	Tat HW+	Kuz HW+	Kuz HW+	 HW+	2.3.i HW+
7				Math Mac	Tat HZ1	Kuz HZ2	Kuz Mac	XXER HW+	XSR2 HW+	XuXR HW+	 HW+	 HW+
8					BurR Mac	Kuz Ea1	Kuz HZ2	2.3.i HW+	 HW+	XXER HW+	 HW+	2.3.i HW+
9						Math ShZ1	XSR2 Ea1	 HW+	 HW+	 HW+		
10							Math Shi2	 HW+	 HW+			2.3.i

References for Table Ia. All upper bounds for $k \geq 4$, $l \geq 6$ were improved in 2019 [AnM2] and 2023 [AnM3], see Table Ib. HW+ abbreviates HWSYZH, as enhanced in [Boza5], see 2.1.m.

k \ l	5	6	7	8	9	10	11	12	13	14	15
4	25	40	58	79	105	135	170	210	256	307	364
5	46	85	133	193	282	381	511	672	860	1081	1341
6		160	270	423	651	944	1346	1855	2499	3301	4305
7			492	832	1368	2119	3197	4665	6653	9260	12635
8				1518	2662	4402	7040	10836			
9					4956	8675	14631				
10						16064					

Table Ib. Upper bounds for $R(k, l)$, $l \geq k \geq 4$, $l \geq 5$. All of them were obtained by Angeltveit and McKay in 2019 [AnM2] and 2023 [AnM3], except $R(4, 5)$ [MR4], and they improve over previously best known bounds reported in Table Ia.

We split the data into Table Ia with a separate table of references corresponding to it, and Table Ib of new upper bounds. In Table Ia, the known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries. For some of the exact values two references are given when the lower and upper bound credits are different. In large computational projects from 2019 to 2023, Angeltveit and McKay run independent computations [AnM2, AnM3], and obtained new upper bounds reported in Table Ib. These, by using the classical recursive upper bound 2.3.a, and the methods of [HW+, HYZ, LiShen], lead to improvements of other higher upper bounds listed in Table Ia but not reported in Table Ib.

(a) The task equivalent to that of proving $R(3, 3) \leq 6$ was the second problem in the Kürschák Mathematics Competitions in Hungary in 1947 [BaLiu]. It also was the second problem in Part I of the William Lowell Putnam Mathematical Competition held in March 1953 [Bush].

(b) Greenwood and Gleason [GG] established the initial values $R(3, 4) = 9$, $R(3, 5) = 14$ and $R(4, 4) = 18$ in 1955.

(c) Kéry [Kéry] proved that $R(3, 6) = 18$ in 1964, but only in 2007 an elementary and self-contained proof of this result appeared in English [Car].

(d) All of the critical graphs for the numbers $R(k, l)$ (graphs on $R(k, l) - 1$ vertices without K_k and without K_l in the complement) are known for $k = 3$ and $l = 3$, 4, 5 [Kéry], 6 [Ka2], 7 [RaK2, McZ], 8 [BrGS] and 9 [GoeR1], and there are 1, 3, 1, 7, 191, 477142, and 1 of them, respectively. There are at least 43×10^6 $(3, 10)$-graphs on 39 vertices [GoeR2, Ang]. All $(3, k)$-graphs, for $k \leq 6$, were enumerated in [RaK2], and all (4,4)-graphs in [MR2]. There exists a unique critical graph for $R(4, 4)$ [Ka2]. Until 2015, there were 350904 known critical graphs for $R(4, 5)$ [MR4], but the full set of such graphs was computed in 2016 [McK3], and there are 352366 of them.

(e) In [MR5], strong evidence is given for the conjecture that $R(5, 5) = 43$ and that there exist exactly 656 critical graphs on 42 vertices. The upper bound of 49 was established in 1997 [MR5]. Angeltveit and McKay improved it to 48 in 2016 [AnM1], and further to 46 in 2023 [AnM3]. It is known that there does not exist any (5, 5)-critical graph which is self-complementary [Chv2].

(f) The graphs constructed by Exoo in [Ex9-Ex25], and some others, are available electronically from http://cs.indstate.edu/ge/RAMSEY. Fujita [Fuj1] maintains a website with some lower bound constructions, using witness graphs obtained independently from Exoo. The equality $R(3,8) = 28$ was verified using a certifiable DRAT proof with SAT + CAS technique [DuLBG]. The equality $R(4,5) = 25$ was verified using the HOL4 (higher order logic) interactive theorem prover system [GauB].

(g) *Cyclic* (or *circulant*) graphs are often used for Ramsey graph constructions. Several cyclic graphs establishing lower bounds were given in the Ph.D. dissertation by J.G. Kalbfleisch in 1966, and many others were published in the next few decades (see [RaK1]). Harborth and Krause [HaKr1] presented all best lower bounds up to 102 from cyclic graphs avoiding complete graphs. In particular, no lower bound in Table Ia can be improved with a cyclic graph on less than 102 vertices, except possibly for $R(3, k)$ for $k \geq 13$. See also items 2.3.l and 5.16.o [HaKr1]. Larger cyclic heuristic constructions for $R(3, k)$ were explored in [JiLTX1, JiLTX2]. Several best lower bounds from *distance colorings*, a slightly more general concept than circular graphs, are presented in [HaKr2].

(h) The claim that $R(5, 5) = 50$ posted on the web [Stone] is in error, and despite being shown to be incorrect more than once, this value is still being cited by some authors. The bound $R(3, 13) \geq 60$ [XieZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw1]. Another incorrect construction for $R(3, 10) \geq 41$ was described in [DuHu].

(i) There are really only two general upper bound inequalities useful for small parameters, namely 2.3.a and 2.3.b. Stronger upper bounds for specific parameters were difficult to obtain, and they often involved massive computations, like those for the cases of (3,8) [McZ], (3,10) [GoeR1, Ang], (4,5) [MR4], (4,6) and (5,5) [MR5, AnM1, AnM2, AnM3]. The bound $R(6,6) \leq 166$, only one more than in [Mac], is an easy consequence of a theorem in [Walk] (2.3.b) and $R(4,6) \leq 41$. Since 2023, we know that $R(6,6) \leq 160$ [AnM3], see Table Ib.

(j) T. Spencer [Spe4], Mackey [Mac], and Huang and Zhang [HZ2], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR3, MR5], were able to establish new upper bounds for several higher Ramsey numbers, improving on all of the previous longstanding best results by Giraud [Gi3, Gi5, Gi6]. These were further improved as reported in Table Ib.

(k) In Table Ia, only some of the higher bounds implied by 2.3.* are shown, and more similar bounds could be derived. In general, we show bounds beyond the contiguous small values if they improve on results previously reported in this survey or published elsewhere. Some easy upper bounds implied by 2.3.a are marked as [Ea1].

(l) In 2009, we have recomputed the upper bounds in Table Ia marked [HZ2] using the method from the paper [HZ2], because the bounds there relied on an overly optimistic personal communication from T. Spencer. Further refinements of this method are studied in [HZ3, ShZ1, Shi2]. The paper [Shi2] subsumes the main results of the manuscripts [ShZ1, Shi2]. All these bounds are now improved by the bounds in Table Ib obtained in [AnM2, AnM3].

(m) In 2013, Boza [Boza5] using the method of [HWSYZH], which is abbreviated as HW+ in Table Ia, computed the bounds marked HW+ by starting from better upper bounds for smaller parameters. Most of the currently shown bounds are thus better than those originally listed in [HWSYZH, HZ3]. All these bounds are now improved by the bounds in Table Ib obtained in [AnM2, AnM3].

(n) In 2015, Exoo and Tatarevic obtained several lower bound improvements marked [ExT] in Tables Ia and IIa by using some modifications of general circulant constructions, but especially related to the quadratic residues Paley graph Q_{101} and the cubic residues graph G_{127}. More bounds by Tatarevic are reported in [Tat]. In 2016, Kuznetsov [Kuz] obtained several further new lower bounds building up on circulant graphs. Also in 2015 and 2016, somewhat surprisingly, Kolodyazhny [Kol1, Kol2] improved four longstanding lower bounds on $R(3, k)$ in Table Ia. The newest improvement of a small lower bound was found by Exoo for $R(5, 6)$ in 2023.

(o) Some lower bounds in Table Ia, like for $R(6,8)$ or $R(8,8)$, may seem rather weak, yet they are not easy to improve. For comments on $R(8,8)$ see [ExT].

(p) A perspective on lower bounds for Ramsey numbers as a statistical physics problem is discussed in [WoGKSF]. It builds up on another approach using adiabatic quantum optimization [RanMCG], but also on more classical heuristic algorithm developed by Exoo in many papers [Ex-nn].

2.2. Bounds for $R(k, l)$, higher parameters

(a) The upper bounds in Tables Ia and IIa marked [GoeR1, Les, Back1] were obtained mainly by deriving lower bounds for several cases of $e(3, k, n)$, which denotes the minimum number of edges in n-vertex triangle-free graphs with independence number less than k. The study of $e(3, k, n)$ was also the main tool for the results obtained in [GrY, GR, RaK2, RaK3, GoeR2].

(b) *Ramsey Calculus* [Back1], is an extensive manuscript by Backelin, which, among other goals, addresses the derivation of $e(3, k, n)$ and the corresponding realisers while avoiding reliance on computer assisted results as far as possible. It achieves the derivation of several lower bounds for $e(3, k+1, n)$ better than those in [GoeR1, RaK3, RaK4] for n close to and above $13k/4$. Better lower bounds on $e(3, k, n)$ sometimes lead to better upper bounds on $R(3, l)$, like for $l = 18$ and $l = 20$ [Back4]. Further improvements to bounds on $e(3, k, n)$ were obtained in [Krü].

k \ l	15	16	17	18	19	20	21	22	23
3	74 Kol2 87 GoeR1	82 Ex21 97 Back3	92 W1+ 109 Back1	99 Ex16 120 Back4	106 W1+ 132 Back3	111 Ex16 145 Les	122 W1+ 157 Back4	131 W2+ 171 Back2	139 XWCS 185 Back2
4	158 Tat	170 Tat	200 Lia+	205 2.3.e	213 2.3.h	234 Ex16	242 SLZL	314 LinCa	
5	275 2.3.h	293 ExT	388 XSR2	396 2.3.h	411 XSR2	424 XSR2	441 2.3.i	492 Ihr	521 2.3.i
6	401 2.3.i	434 SLLL	548 SLLL	614 SLLL	710 SLLL	878 SLLL	888 2.3.h	1070 SLLL	
7		629 2.3.i	729 2.3.i	797 2.3.i	908 SLLL		1214 SLLL		
8	873 2.3.i		1005 2.3.i	1049 2.2.h	1237 2.2.h		1617 2.3.i		

Table IIa. Known bounds for higher two-color Ramsey numbers $R(k, l)$, with references.
Lower and upper bounds are given for $k = 3$, only lower bounds for $k \geq 4$;
Lia+, W1+ and W2+ abbreviate LiaWXS, WWY1 and WSLX2, respectively.

k \ l	24	25	26	27	28	29	30	31	32
3	143 W1+	154 W2+	161 FuLS	172 LiLi	179 FuLS	190 LiLi	197 FuLS	208 FuLS	217 LiLi

k \ l	33	34	35	36	37	38	39	40	41
3	227 LiaX	234 LiaX	248 LiaX	255 FuLS	267 LiaX	278 LiaX	290 LiaX	298 LiaX	311 FuLS

k \ l	42	43	44	45	46	47	48	49	50
3	320 FuLS	333 FuLS	339 FuLS	354 Ji+	362 FuLS	380 Ji+	384 Ji+	402 Ji+	

Table IIb. Known lower bounds for higher Ramsey numbers $R(3, l)$ for $l \geq 24$.
W1+, W2+ and Ji+ abbreviate WSLX1, WSLX2 and JiLTX2, respectively.

k	11	12	13	14	15	16	17
lower bound	1597	1640	2557	2989	5485	5605	8917
reference	2.2.c	Tat	2.2.c	2.2.c	2.2.c	2.2.c	LuSL
k	18	19	20	21	22	23	24
lower bound	11005	17885	21725	30925	39109	49421	
reference	LuSL	LuSL	Ex23	Ex23	Ex23	Ex23	

Table IIc. Known lower bounds for diagonal Ramsey numbers $R(k, k)$ for $k \geq 11$; All lower bounds for $k \geq 13$ are from Paley graphs, see also 2.2.c below.

(c) The construction by Mathon (discovered later but independently by Shearer [She2], see also items 2.3.j, 6.2.k and 6.2.l), using the data obtained by Shearer [She4] for primes up to 7000, implies the lower bounds in Table IIc marked 2.2.c. The first two bounds credited in Table IIc to [LuSL] also follow similarly from the data in [She4]. The same approach does not improve on the bound $R(12, 12) \geq 1639$ [XSR2], later increased to 1640 [Tat]. The bounds in [Ex23] were obtained by extending data for Paley graphs beyond [Sha4] and improving on [LiaWXCS].

(d) The lower bounds marked [XuXR], [XXER], [XSR2], 2.3.e, 2.3.h and 2.3.i need not be cyclic. Several of the Cayley colorings from [Ex16] are also non-cyclic. All other lower bounds listed in Table IIa/b were obtained by construction of circular graphs.

(e) The graphs establishing lower bounds marked 2.3.h can be constructed by using appropriately chosen graphs G and H with a common m-vertex induced subgraph, similarly as it was done in several cases in [XuXR].

(f) Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing several lower bounds for $R(3, k)$, for $k \geq 61$. All of these bounds can be improved by the inequalities in 2.3.c and data from Tables Ia and IIa/b.

(g) Unpublished bound $R(4, 22) \geq 314$ [LiSLW] improved over 282 given in [SuL]. [LinCa] obtained the same bound, and also $R(4, 25) \geq 458$. Not yet published bounds $R(3, 23) \geq 139$ [XWCS] and $R(4, 17) \geq 200$ [LiaWXS] improve over 137 and 182 obtained in [WSLX2] and [LuSS1], respectively. The bound $R(9, 17) \geq 1411$ is given in [XuXR]. Large cyclic heuristic constructions for $R(3, k)$ for $k < 50$ were explored in [JiLTX1, JiLTX2].

(h) Two special cases, $R(8, 18) \geq 1049$ and $R(8, 19) \geq 1237$, can be obtained by applying 2.3.i and 2.3.h below. In both cases we start with the 816-vertex graph G, witnessing $R(8, 13) \geq 817$, obtained by 2.3.i. Next, for properly chosen graphs H in the application of 2.3.h, we have large common subgraphs of G and H, namely the 101-vertex witness of $R(6, 6) \geq 102$ and the 204-vertex witness of $R(7, 7) \geq 205$, respectively.

(i) One can expect that the lower bounds in Tables IIa/b are weaker than those in Table Ia, especially smaller ones, in the sense that some of them should not be that hard to improve, in contrast to the bounds in Table Ia.

2.3. General results on $R(k, l)$

(a) $R(k, l) \le R(k-1, l) + R(k, l-1)$, with strict inequality when both terms on the right hand side are even [GG]. There are obvious generalizations of this inequality for avoiding graphs other than complete.

(b) $R(k, k) \le 4R(k, k-2) + 2$ [Walk].

(c) Explicit construction for $R(3, 3k+1) \ge 4R(3, k+1) - 3$, for all $k \ge 2$ [CleDa], explicit construction for $R(3, 4k+1) \ge 6R(3, k+1) - 5$, for all $k \ge 1$ [ChCD].

(d) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CoPR]. For other constructive results in relation to $R(3, k)$ see [BrBH1, BrBH2, Fra1, Fra2, FrLo, GoeR1, Gri, KlaM1, Loc, RaK2, RaK3, RaK4, Stat, Yu1]. See also 2.4.3 and 2.4.4 in the next subsection.

(e) The study of bounds for the difference between consecutive Ramsey numbers was initiated in [BEFS], where the bound $R(k, l) \ge R(k, l-1) + 2k - 3$, for $k, l \ge 3$, was established by a construction. In 1980, Erdős and Sós (cf. [Erd2,ChGra2]) asked: If we set $\Delta_{k,l} = R(k, l) - R(k, l-1)$, then is it true that $\Delta_{k,k+1}/k \to \infty$ as $k \to \infty$? Only easy bounds on $\Delta_{k,l}$ are known, in particular for $k = 3$ we have $3 \le \Delta_{3,l} \le l$. For some discussion of the roadblocks on the latter see [XSR2, GoeR2, ZhuXR]. It is also known that $R(3, k) \ge R(3, K_{k-1} - e) + 4$ [ZhuXR].

(f) A conjecture that $R(k, l) \ge R(k-1, l+1)$, for all $3 \le k \le l$, is dubbed DC for the Diagonal Conjecture. Its implications, evidence for validity, and some related problems are discussed in [LiaRX]. For the multicolor version of the DC and its consequences see item 6.2.v.

(g) By taking a disjoint union of two critical graphs one can easily see that $R(k, p) \ge s$ and $R(k, q) \ge t$ imply $R(k, p+q-1) \ge s+t-1$. Xu and Xie [XuX1] improved this construction to yield better general lower bounds, in particular $R(k, p+q-1) \ge s+t+k-3$.

(h) For $2 \le p \le q$ and $3 \le k$, if (k, p)-graph G and (k, q)-graph H have a common induced subgraph on m vertices without K_{k-1}, then $R(k, p+q-1) > n(G) + n(H) + m$. In particular, this construction implies the bounds $R(k, p+q-1) \ge R(k, p) + R(k, q) + k - 3$ and $R(k, p+q-1) \ge R(k, p) + R(k, q) + p - 2$ [XuX1, XuXR], with some small improvements, such as using the term $k-2$ instead of $k-3$ in the first bound for $k \ge 5$ [XSR2].

(i) $R(2k-1, l) \ge 4R(k, l-1) - 3$ for $l \ge 5$ and $k \ge 2$, and in particular for $k = 3$ we have $R(5, l) \ge 4R(3, l-1) - 3$ [XXER].

(j) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no K_k, then $R(k, k) \ge p+1$ and $R(k+1, k+1) \ge 2p+3$ [Math, She2]. Data for larger p was obtained in [LuSL], and further for p up to 25000 in [Ex23]. See also 3.1.f, and items 6.2.k and 6.2.l for similar multicolor results.

(k) Study of Ramsey numbers for large disjoint unions of graphs [Bu1, Bu9], in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$, for n large enough [Bu8].

(l) $R(k, l) \ge L(k, l) + 1$, where $L(k, l)$ is the maximal order of any cyclic (k, l)-graph. A compilation of many best cyclic bounds was presented in [HaKr1].

(m) The graphs critical for $R(k, l)$ are $(k-1)$−vertex connected and $(2k-4)$−edge connected, for $k, l \geq 3$ [BePi]. This was improved to vertex connectivity k for $k \geq 5$ and $l \geq 3$ in [XSR2].

(n) All Ramsey-critical (k, l)−graphs are Hamiltonian for $k \geq l-1 \geq 1$ and $k \geq 3$, except when $(k, l) = (3, 2)$ [XSR2].

(o) Two-color lower bounds can be obtained by using items 6.2.m, 6.2.n and 6.2.o with $r = 2$. Some generalizations of these were obtained in [ZLLS].

2.4. Some pillars of asymptotics of $R(k, l)$

In this section we give only some selected pointers to the literature dealing with asymptotics of two-color Ramsey numbers. This survey was designed mostly for small, finite, and combinatorial results, but still we wish to give the reader some useful and representative references to more traditional papers studying the infinite. In particular, we include the main breakthroughs in this area, such as the progress on asymptotics of $R(3, k)$, $R(4, k)$ and $R(k, k)$.

(1) In 1947, Erdős gave a simple probabilistic proof that $R(k, k) > 2^{k/2}$ [Erd1]. In 1975, Spencer [Spe1] improved it to $R(k, k) > \sqrt{2}e^{-1}k2^{k/2}(1+o(1))$. More probabilistic asymptotic lower bounds were obtained in [Spe1, Spe2, AlPu].

(2) The limit of $R(k, k)^{1/k}$, if it exists, is between $\sqrt{2}$ and 4 [GRS, GrRö, ChGra2]. This limit is now known to be strictly below 4, as implied by the last result of item (8) below.

(3) In 1995, Kim obtained a breakthrough result by proving that $R(3, k) = \Theta(k^2/\log k)$ [Kim]. The best known lower and upper bounds constants are 1/4 [BohK2, BohK3] and 1 (implicit in [She1]), respectively. The optimality of the constant 1/4 is conjectured in [BohK3]. An independent proof of the lower bound constant 1/4 and a conjecture that it is best possible are presented in [FizGM].

(4) Other asymptotic and general results on triangle-free graphs in relation to $R(3, k)$ can be found in [Boh, AlBK, AjKS, Alon2, CleDa, ChCD, CoPR, Gri, FrLo, Loc, She1, She3].

(5) Explicit constructions yielded the lower bounds $R(4, k) \geq \Omega(k^{8/5})$, $R(5, k) \geq \Omega(k^{5/3})$ and $R(6, k) \geq \Omega(k^2)$ [KosPR]. For the same cases of k classical probabilistic arguments give $\Omega((k/\log k)^{5/2})$, $\Omega((k/\log k)^3)$ and $\Omega((k/\log k)^{7/2})$, respectively [Spe2]. These were improved to $\Omega(k^{5/2}/(\log k)^2)$, $\Omega(k^3/(\log k)^{8/3})$ and $\Omega(k^{7/2}/(\log k)^{13/4})$, respectively, in [Boh, BohK1], and in general to $R(s, t) \geq \Omega(t^{(s+1)/2}/(\log t)^{(s^2-s-4)/(2s-4)})$, for fixed s and large t [BohK1].

(6) In 2023 arXiv posting (journal paper appeared in 2024), Mattheus and Verstraëte [MatVer] proved that there exist constants c_1 and c_2 such that for all $k \geq 3$ we have

$$c_1 k^3/\log^4 k \leq R(4, k) \leq c_2 k^3/\log^2 k.$$

A brief background and explanation of this breakthrough result, and its link to finite geometry, are presented in [Bish].

(7) Explicit construction of a graph with clique and independence k on $2^{c\log^2 k/\log\log k}$ vertices was presented by Frankl and Wilson [FraWi], and further constructions by Chung [Chu3] and Grolmusz [Grol1, Grol2]. In 2012, the best explicit construction for large k by Barak et al. [BarRSW] improved over [FraWi] by giving such a graph on $2^{2^{(\log\log k)^c}}$ vertices for some $c > 1$, or equivalently, on n vertices, where $\log\log n = (\log\log k)^c$. In 2016, this was improved by Cohen [Coh] to $\log\log n = (\log k)^d$, for a positive constant d, and in a 2023 arXiv posting, Xin Li [LiXin] presents explicit Ramsey graphs on n vertices with no clique or independent set of order $\log^c n$, for some $c > 1$. Explicit constructions such as these are usually weaker than known probabilistic results.

(8) In 2009, Conlon [Con1] obtained the best until then upper bound for the diagonal case

$$R(k+1,\ k+1) \leq \binom{2k}{k} k^{-c\log k/\log\log k}.$$

In 2020 on arXiv (journal paper appeared in 2023), Sah [Sah] improved it to

$$R(k+1,\ k+1) \leq \binom{2k}{k} e^{-c(\log k)^2}.$$

In 2023, Campos, Griffiths, Morris and Sahasrabudhe [CamGMS] posted a 57-page long paper on arXiv showing that there exists $\varepsilon>0$ such that

$$R(k,\ k) \leq (4-\varepsilon)^k.$$

Thus, in (2), the upper bound on the limit $R(k,\ k)^{1/k}$ is $4-\varepsilon$.

(9) Other older asymptotic bounds can be found, for example, in [Chu3, McDS, Boh, BohK1] (lower bound) and [Tho] (upper bound), and for many other bounds in the general case of $R(k,\ l)$ consult [Spe2, GRS, GrRö, Chu4, ChGra2, LiRZ1, AlPu, Kriv, ConFS7]. Two nontechnical essays by Sloman on recent developments in this area appeared in the *Quanta Magazine* in 2023 [Slo].

3. Two Colors: $K_n - e$, K_3, $K_{m,n}$

3.1. Dropping one edge from complete graph

This section contains known values and nontrivial bounds for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete.

(a) The exact values in Table IIIa involving $K_3 - e$ are obvious, since one can easily see that $R(K_3 - e, K_k) = R(K_3 - e, K_{k+1} - e) = 2k - 1$ for all $k \geq 2$.

(b) More bounds (beyond those shown in Tables IIIa/b) can be easily obtained using Table Ia/b, an obvious generalization of the inequality $R(k, l) \leq R(k-1, l) + R(k, l-1)$, and by the monotonicity of Ramsey numbers, in this case $R(K_{k-1}, G) \leq R(K_k - e, G) \leq R(K_k, G)$.

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3-e	3	5	7	9	11	13	15	17	19
K_3	5	7	11	17	21	25	31	37	42 45
K_4-e	5	10	13	17	28	30 32	36 45	43 57	 73
K_4	7	11	19	30	37 49	52 71	62 102	 135	 170
K_5-e	7	13	22	37	65	66 91	69 136	 188	 261
K_5	9	16	30 33	43 62	65 102	81 173	121 262	 381	 511
K_6-e	9	17	37	45 70	66 124	83 206	 334	 505	 757
K_6	11	21	43 53	58 104	 205	 353	 612	 944	 1346
K_7-e	11	28	65	66 124	 247	 432	 761	 1218	 1964
K_7	13	28 29	65 82	80 184	 370	 716	 1269	 2119	 3197
K_8	15	36 39	69 120	 286	 646	 1281	 2518	 4402	 7040
K_9	17	41 53	75 172	 456	 1072	 2340	 4686	 8675	 14631
K_{10}	19	49 68	 236	 666	 1702	 3880	 8413	 16064	

Table IIIa. Bounds on the Ramsey numbers $R(G, H)$, for complete or missing one edge graphs G and H, but not both complete. Known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

G \ H	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$	$K_{11}-e$
K_3	ChH2	Clan	FRS1	GrH	Ra1	Ra1	MPR GoeR2	WWY2 GoeR2
K_4-e	ChH1	FRS2	McR	McR	AnM2 LidP	VO BZ2	GoeVO BZ2	BZ2
K_4	ChH2	EHM1	Boza6 JamKR	Ex14 LidP	VO BZ3	VO Ea1	AnM3	AnM3
K_5-e	FRS2	CE+	VO	VO	Ea1 BZ3	Ea1 Ea1	BZ3	BZ3
K_5	BoH	Ex6 Boza7	Ea1 LidP	VO LidP	VO BZ3	VO BZ3	AnM3	AnM3
K_6-e	McR	VO	Ex14 HZ3	VO LidP	VO BZ3	BZ3	BZ3	BZ3
K_6	McN/ ShWR	VO BZ1	Ea1 BZ4	ShZ2	BZ4	BZ3	AnM3	AnM3
K_7-e	McR	VO	VO LidP	Ea1	BZ3	BZ3	BZ3	BZ3
K_7	Ea1 LidP	Ea1 Ea1	Ea1 BZ3	BZ4	BZ4	BZ4	AnM3	AnM3
K_8	VO LidP	Ea1 BZ3	BZ3	BZ3	BZ4	BZ4	AnM3	AnM3
K_9	VO BZ2	Ea1 BZ3	BZ3	BZ3	BZ3	BZ4	AnM3	AnM3
K_{10}	VO BZ2	BZ3	BZ3	BZ3	BZ3	BZ3	AnM3	

References for Table IIIa.
CE+ abbreviates ClEHMS, for some details of BZ1-BZ4 see item 3.1.g, the bounds marked [AnM3] are trivially implied by entries in Table Ib.

k	11	12	13	14	15	16
lower bound	42 WWY2	49 VO	55 GoeR2	61 VO	69 WWY2	74 Ea1
upper bound	45 GoeR2	53 GoeR2	62 GoeR2	71 GoeR2	80 GoeR2	91 GoeR2

Table IIIb. Lower and upper bounds for $R(K_3, K_k - e)$ for $11 \le k \le 16$.

(c) Upper bounds for Ramsey numbers $R(K_k, K_l - e)$ marked [AnM3] in references for Table IIIa are trivially implied by the bounds on $R(K_k, K_l)$ in Table Ib. These were obtained in 2023 by Angeltveit and McKay using linear programming, and they improve over upper bounds in Table Ia. [AnM2] points to little older and weaker bounds for some higher cases. The upper bounds by Lidický and Pfender [LidP] use flag algebras.

(d) Two special exact values, and several other bounds were obtained by Van Overberghe [VO] in 2020. The surprisingly large exact values $R(K_5-e, K_6-e) = 37$ and $R(K_5-e, K_7-e) = 65$ exploit some previously known strongly regular graphs on 27, 36 and 64 vertices, namely the Schläfli graph, $NO^{-}(6,2)$ and $VO^{-}(6,2)$. See the website by A. E. Brouwer [Brou] for a great collection of strongly regular graphs, including those used in [VO].

(e) Some lower bounds beyond the range of Table IIIa are as follows:

$49 \le R(K_3, K_{12}-e)$	[GoeVO]
$61 \le R(K_3, K_{14}-e)$	[GoeVO]
$82 \le R(K_4-e, K_{16}-e)$	[VO]
$128 \le R(K_4, K_{12}-e)$	[Shao]
$2987 \le R(K_{14}-e, K_{14}-e)$	[LiShen]

The results of MS thesis [VO] are contained and extended in a journal article [GoeVO]. The bound $R(K_4, K_{12}-e) \ge 128$ [Shao] is derived by using one color of the (4, 4, 4;127)-coloring defined in [HiIr]. For comments related to the bound in [LiShen], see the next item.

(f) If the quadratic residues Paley graph Q_p of prime order $p = 4t+1$ contains no $K_k - e$, then $R(K_{k+1}-e, K_{k+1}-e) \ge 2p+1$. In particular, $R(K_{14}-e, K_{14}-e) \ge 2987$ [LiShen]. This was generalized to $K_k - F$ for some small graphs F instead of an edge $e\,(=K_2)$ [WaLi]. See also item 2.3.j.

(g) This item follows from personal communications by Boza [Boza5, Boza8], The upper bound marked [BZ1] was obtained in 2012, while [BZ2, BZ3, BZ4] are from 2024, all by Boza. [BZ2] refers to computer-free simple proofs of some upper bounds, those marked [BZ3] use the methods and theorems in [LiShen, HYZ, HWSYZH], and [BZ4] use in addition item 5.14.n. These approaches expand on 2.1.m, or are implied by [Boza6], the previous work [Boza1, Boza3, BoPo], the method of [HZ3], and the bounds given in [GoeR2, AnM2, AnM3]. The enumeration of all (K_6, K_4-e)-graphs [ShWR] is used in [BoPo].

(h) All (K_3, K_k-e)-graphs were enumerated for $k \le 6$ [Ra1] and $k = 7$ [Fid2, GoeR2]. Full sets of (K_l, K_k-e)-graphs were posted for the parameters (K_3, K_k-e) for $k \le 7$, (K_4, K_k-e) for $k \le 5$, and (K_5, K_k-e) for $k \le 4$ ([Fid2], available until 2014), and other full and restricted families at [BrCGM, Fuj1].

(i) The number of (K_3, K_k-e)-critical graphs for $k = 4$, 5 and 8 is 4, 2 and 9, respectively [MPR]. There are 7 critical graphs for $R(K_3, K_9-e)$, and at least 40 such graphs for $R(K_3, K_{10}-e)$ [GoeR2].

(j) The critical graphs are unique for: $R(K_3, K_k-e)$ for $k=3$ [Tr], 6 and 7 [Ra1], $R(K_4-e, K_4-e)$ [FRS2], $R(K_5-e, K_5-e)$ [Ra3, GoeVO] and $R(K_4-e, K_7-e)$ [McR]. All of the critical graphs for the cases $R(K_4-e, K_4)$ [EHM1], $R(K_4-e, K_5)$ and $R(K_5-e, K_4)$ [DzFi1] are known, and there are 5, 13 and 6 of them, respectively. The unpublished value of $R(K_4-e, K_6)$ [McN] was confirmed in [ShWR], where in addition all 24976 critical graphs were found. All (K_4-e, K_6-e)-graphs were enumerated in [GoeVO].

(k) If $m \le n$ then $R(K_4 - e, K_{m+n+1}) \ge R(3, m+1) + R(3, n+1) + n$.
Study of the growth of $R(K_4 - e, K_n)$ and its relationship to $R(K_3, K_n)$ [JiLSX].

(l) $R(K_k - e, K_k - e) \le 4R(K_{k-2}, K_k - e) - 2$ [LiShen].
For a similar inequality for complete graphs see 2.3.b.

(m) Study of the cases $R(K_m, K_n - K_{1,s})$ and $R(K_m - e, K_n - K_{1,s})$, with several exact values for special parameters [ChaMR]. This study was extended to some cases involving $R(K_m - K_3)$ [MonCR].

(n) The upper bounds from [ShZ2] are subsumed by a later article [Shi2].

(o) The upper bounds in [HZ3] were obtained by a reasoning generalizing the bounds for classical numbers in [HZ2]. Several other results from Section 2.3 apply, though checking in which situation they do may require looking inside the proofs whether they still hold for $K_n - e$. The upper bounds in the manuscript [HTHZ1] are based on [HZ3].

3.2. Triangle versus other graphs

(a) $R(3, k) = \Theta(k^2/\log k)$ [Kim].
For more comments on asymptotics see Section 2.3.(3) and the items 3.2.p/q below.

(b) Explicit construction for $R(3, 3k+1) \ge 4R(3, k+1) - 3$, for all $k \ge 2$ [CleDa],
explicit construction for $R(3, 4k+1) \ge 6R(3, k+1) - 5$, for all $k \ge 1$ [ChCD].

(c) Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2, CoPR].

(d) $R(K_3, K_7 - 2P_2) = R(K_3, K_7 - 3P_2) = 18$ [SchSch2].

(e) $R(K_3, K_3 + \overline{K}_m) = R(K_3, K_3 + C_m) = 2m + 5$, for $m \ge 212$ [Zhou1].

(f) $R(K_3, K_2 + T_n) = 2n + 3$ for n-vertex trees T_n, for $n \ge 4$ [SonGQ],
$R(K_3, K_1 + nK_3) = 6n + 1$, for $n \ge 3$ [HaoLin].

(g) $R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G)+1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \ge 4$ [BEFRS1].

(h) $R(K_3, Q_n) = 2^{n+1} - 1$ for large n [GrMFSS], where Q_n is the n-dimensional hypercube. For related publications on the general case of $R(K_m, Q_n)$ see [FizGMSS, ConFLS] and item 5.15.n.

(i) Relations between $R(3, k)$ and graphs with large $\chi(G)$ [BiFJ],
further detailed study of the relation between $R(3, k)$ and the chromatic gap [GySeT].

(j) $R(K_3, G) \le 2e(G) + 1$ for any graph G without isolated vertices [Sid3, GodK].

(k) $R(K_3, G) \le n(G) + e(G)$ for all G, a conjecture [Sid2].

(l) $R(K_3, G)$ for all connected G up to 9 vertices [BrBH1, BrBH2].

(m) $R(K_3, G)$ for all graphs G on 10 vertices [BrGS], except 10 cases (three of which, including $G = K_{10} - e$, were solved [GoeR2]). See also several items in Section 8.1.

(n) $R(nK_3, nK_3) = 5n$ for $n \geq 2$, $R(mK_3, nK_3) = 3m + 2n$ for $m \geq n \geq 2$ [BES], and $R(c(nK_3), c(nK_3)) = 7n - 2$ for $n \geq 2$, where $c(nK_3)$ is any connected graph containing n vertex disjoint triangles [GySá3].

(o) Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng].

(p) For every positive constant c, and for Δ and n large enough, there exists n-vertex graph G with $\Delta(G) \leq \Delta$ for which $R(K_3, G) > cn$ [Bra3].

(q) $R(K_3, K_{k,k}) = \Theta(k^2/\log k)$ [LinLi2].

(r) For $R(K_3, K_n)$ see Section 2, and for $R(K_3, K_n - e)$ see Section 3.1.

(s) Since $B_1 = F_1 = C_3 = W_3 = K_3$, other sections apply. For some other cases involving triangle see also [Boh, AjKS, BrBH1, BrBH2, FrLo, Fra1, Fra2, BiFJ, Gri, GySeT, Loc, KlaM1, LiZa2, RaK2, RaK3, RaK4, She1, She3, Spe2, Stat, Yu1].

3.3. Complete bipartite graphs

This subsection gathers information on Ramsey numbers where specific bipartite graphs are avoided in edge colorings of K_n (as everywhere in this survey), in contrast to the often studied bipartite Ramsey numbers, which are not covered in this survey, where the edges of complete bipartite graphs $K_{n,m}$ are colored.

3.3.1. Numbers

n	$R(C_4, K_{1,n})$	reference	n	$R(C_4, K_{1,n})$	reference
2	4	ChH2	22	28	SunSh
3	6	ChH2	23	29	Par5
4	7	Par3	24	30	WuSZR
5	8	Par3	25	31	Par3
6	9	FRS4	26	32	Par3
7	11	FRS4	27	33	Boza8
8	12	Tse1	28	35	Boza8
9	13	Par3/Tse1	29	36	Boza8
10	14	Par3/Tse1	30	36-37	WuSR/DyDz2
11	16	Tse1	31	38	Boza8
12	17	Tse1	32	39	Boza8
13	18	DyDz2	33	39-40	WuSR/DyDz2
14	19	DyDz2	34	41	WuSR
15	20	Law2/DyDz2	35	42	WuSR
16	21	Par3/DyDz2	36	43	WuSR
17	22	Par3	37	44	Boza8
18	23	ZhaBC1	38	45	ZhaCC2
19	24	WuSR	39	46-47	WuSR/DyDz2
20	25	WuSR	40	47	ZhaCC2
21	27	Par5	41	49	Boza8

Table IVa. Values and bounds for Ramsey numbers $R(C_4, K_{1,n})$ for $n \leq 41$, $C_4 = K_{2,2}$.

Table IVa presents data for $C_4 = K_{2,2}$ versus stars, while the following Tables IVb and IVc gather information about small complete bipartite graphs. Their first versions were based on the surveys by Lortz and Mengersen [LoM3, LoM4]. All cases involving $K_{1,2} = P_3$ are solved by a formula for $R(P_3, G)$, which holds for all isolate-free graphs G, derived in [ChH2]. The star versus star numbers are given below in the item 3.3.2.a and in Section 5.5.

(a) Note that for graph G to avoid $K_{1,n}$ is equivalent to $\delta(G) < n$. Thus, for general monotonicity we have, for example, that rows of Table IVb are nondecreasing, but we do not know if they are strictly increasing.

(b) See function $f(n)$ in the item 3.3.2.c as one involving C_4. Similarly, we also have $R(C_4, K_{1,n+1}) \leq R(C_4, K_{1,n}) + 2$ [Chen], which is the same claim as in 3.3.2.d.

(c) $R(C_4, K_{1,n}) = R(C_4, W_{n+1})$ for $n \geq 6$ [ZhaBC1]. See also items (b)-(f) in Section 4.3.2 concerning results on Ramsey numbers of wheels versus C_4. Values and bounds on several cases higher than those in Table IVa are reported in [NoBa, WuSR].

(d) For all odd prime powers q we have [Boza8]:

$$q^2+1 \leq R(C_4, K_{1,q^2-q+1}) \leq q^2+2 \leq R(C_4, K_{1,q^2-q+2}) \leq q^2+3.$$

A number of similar results were included in the early papers by Parsons [Par3, Par5], and many in newer papers such as [ZhaBC1, WuSR, WuSZR, ZhaCC2, DyDz2].

(e) Formula for $R(K_{1,n}, K_{k_1, k_2, \cdots, k_t, m})$ for m large enough, in particular for $t=1$, $k_1=2$ with $n \leq 5$, $m \geq 3$ and $n=6$, $m \geq 11$, for example $R(K_{1,5}, K_{2,7}) = 15$ [Stev].

(f) The values and bounds for higher cases of $R(K_{2,2}, K_{2,n})$ are 20, 22, 22, 24, 25, 26, 27/28, 28/29, 30 and 32 for $12 \leq n \leq 21$, respectively. All of them were given in [HaMe4], except those for n = 14, 15 and 18, which were obtained in [Dyb1]. More exact values for prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$ can be found in [HaMe4].

(g) The known values of $R(K_{2,2}, K_{3,n})$ are 15, 16, 17, 20 and 22 for $6 \leq n \leq 10$ [Lortz], and $R(K_{2,2}, K_{3,12}) = 24$ [Shao]. See Tables IVa and IVb for the smaller cases, and [HaMe4] for upper bounds and values for some prime powers $\lceil \sqrt{n} \rceil$.

(h) $R(K_{2,n}, K_{2,n})$ is equal to 46, 50, 54, 57 and 62 for $12 \leq n \leq 16$, respectively.
The first open diagonal case is $65 \leq R(K_{2,17}, K_{2,17}) \leq 66$ [EHM2].
The status of all higher cases for $n < 30$ is listed in [LoM1].

m, n \ p, q	1, 2	1, 3	1, 4	1, 5	1, 6	2, 2	2, 3	2, 4	2, 5	3, 3	3, 4
2, 2	4 ChH2	6 ChH2	7 Par3	8 Par3	9 FRS4	6 ChH1					
2, 3	5 ChH2	7 FRS4	9 Stev	10 FRS4	11 FRS4	8 HaMe4	10 Bu4				
2, 4	6 ChH2	8 HaMe3	9 Stev	11 HaMe4	13 LoM4	9 HaMe4	12 ExRe	14 EHM2			
2, 5	7 ChH2	9 HaMe3	11 Stev	13 Stev	14 LoM4	11 HaMe4	13 LoM3	16 LoM1	18 EHM2		
2, 6	8 ChH2	10 HaMe3	11 Stev	14 Stev	15* Shao	12 HaMe4	14 LoM3	17 LoM3	20 LoM1		
3, 3	7 ChH2	8 HaMe3	11 LoM4	12 LoM4	13 LoM4	11 Lortz	13 HaMe3	16 LoM4	18 LoM4	18 HaMe3	
3, 4	7 ChH2	9 HaMe3	11 LoM4	13 LoM4	14 LoM4	11 Lortz	14 LoM4	17 Sh1+	20 VO-LidP	19-20 VO-LidP	25 VO-LidP
3, 5	9 ChH2	10 HaMe3	13 Sh1+	15 Sh1+	17 LidP	14 HaMe4	17* Shao	19-20 VO-LidP	21-23 VO-LidP	21-24 VO-LidP	25-29 VO-LidP

Table IVb. Ramsey numbers $R(K_{m,n}, K_{p,q})$; unpublished result marked with *, Sh1+ abbreviates ShaXBP, the bound $R(K_{3,5}, K_{2,5}) \geq 21$ is also in [ShaoWX].

n \ m	2	3	4	5	6	7	8	9	10	11
6	12 HaMe4	14 LoM3	17 LoM3	20 LoM1	21 EHM2					
7	14 HaMe4	17 LoM3	19 LoM3	21 LoM3	24 LoM1	26 EMH2				
8	15 HaMe4	18 LoM3	20 LoM3	22-23 LoM3	25 VO-LoM3	28 LoM1	30 EMH2			
9	16 HaMe4	19 LoM3	22 LoM3	25* Shao	27* Shao	29* Shao	32 LoM1	33 EHM2		
10	17 HaMe4	21 LoM3	24 LoM3	27 LoM3	27-29 LoM3	29-31 VO-LoM3	33 VO-LoM3	36 LoM1	38 EHM2	
11	18 HaMe4	≥22 VO	≥25 VO	≥28 VO	≥29 VO	≥33 VO	35 VO-LoM3	37 VO-LoM3	40 LoM1	42 EHM2

Table IVc. Known Ramsey numbers $R(K_{2,n}, K_{2,m})$ for $6 \leq n \leq 11$ and $2 \leq m \leq 11$. Results marked [VO] are from MS thesis by Van Overberghe, and they now appeared in a journal paper [GoeVO]. Unpublished results improving over [LoM3] are marked with a *.

(i) Some exact values and bounds for parameters beyond the range of Tables IVb/c are:

$R(K_{1,4}, K_{4,4}) = R(K_{1,5}, K_{4,4}) = 13$	[ShaXPB]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$	[GuSL]
$R(K_{1,7}, K_{2,3}) = 13$	[Par4, Par6]
$R(K_{2,2}, K_{4,4}) = 14$	[HaMe4]
$R(K_{2,2}, K_{4,5}) = 15$	[Shao]
$R(K_{2,2}, K_{4,6}) = 16$	[Shao]
$R(K_{2,2}, K_{5,5}) = R(K_{2,3}, K_{3,5}) = 17$	[Shao]
$R(K_{3,5}, K_{3,5}) = 33$	[VO][LidP]
$33 \le R(K_{4,4}, K_{4,4}) \le 49$	[VO][LidP]

(j) A number of general upper and lower bounds for $R(K_{s,t}, K_{s,t})$, in particular for small fixed s, and for some slightly off-diagonal cases were obtained in [LoM2]. They can be used to derive the upper bounds for the cases listed in (i) above and (k) below.

(k) Several lower bounds of the form $R(K_{s,t}, K_{s,t}) \ge m$ from distance colorings, a slightly more general concept than circular graphs, were presented in [HaKr2] for the following triples (s, t, m): (3,6,38), (3,7,42), (3,8,43), (3,9,54), (4,5,42), (4,6,43), (4,7,54), (5,5,54).

3.3.2. General results

(a) $R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Har1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

(b) $R(K_{1,3}, K_{m,n}) = m + n + 2$ for $m, n \ge 1$ [HaMe3].

(c) $R(K_{1,n}, K_{2,2}) = f(n) \le n + \lceil \sqrt{n} \rceil + 1$, with $f(q^2) = q^2 + q + 1$ and $f(q^2 + 1) = q^2 + q + 2$ for every q which is a prime power [Par3]. Furthermore, $f(n) \ge n + \sqrt{n} - 6n^{11/40}$ [BEFRS4]. For more bounds on $f(n)$ see [Par5, Chen, ChenJ, MoCa, WuSR, WuSZR, ZhaBC1, BoRa]. Summary of what is known and further progress are reported in two 2017 papers [ZhaCC2, ZhaCC3]. With $f(29) = 36$ obtained in [Boza8], the values of $f(n)$ are known for all $n \le 29$. Compare also to items in Section 4.3 involving C_4.

(d) $R(K_{1,n+1}, K_{2,2}) \le R(K_{1,n}, K_{2,2}) + 2$ [Chen].

(e) $R(K_{2,\lambda+1}, K_{1,v-k+1})$ is either $v + 1$ or $v + 2$ if there exists a (v, k, λ)-difference set. This and other related results are presented in [Par4, Par5]. See also [GoCM, GuLi].

(f) Formulas and bounds on $R(K_{2,2}, K_{2,n})$, and bounds on $R(K_{2,2}, K_{m,n})$. In particular, we have $R(K_{2,2}, K_{2,k}) = n + k\sqrt{n} + c$, for $k = 2, 3, 4$, some prime powers $\lceil \sqrt{n} \rceil$ and $\lceil \sqrt{n} \rceil + 1$, and some $-1 \le c \le 3$ [HaMe4]. An improvement of the latter for some special cases of n was obtained in [Dyb1]. Asymptotics of $R(K_{2,2}, K_{n,n})$ is discussed in [LiuLi2], where in particular the lower bound $R(K_{2,2}, K_{n,n}) = \Omega(n^{3/2}/\log n)$ is presented. See also item 4.2.d.

(g) $R(K_{2,n}, K_{2,n}) \le 4n - 2$ for all $n \ge 2$, and the equality holds if and only if there exists a strongly regular $(4n - 3, 2n - 2, n - 2, n - 1)$-graph [EHM2].

(h) Conjecture that $4n-3 \le R(K_{2,n}, K_{2,n}) \le 4n-2$ for all $n \ge 2$. Many special cases are solved and several others are discussed in [LoM1].

(i) $R(K_{2,n-1}, K_{2,n}) \le 4n-4$ for all $n \ge 3$, with the equality if there exists a symmetric Hadamard matrix of order $4n-4$. There are only 4 cases in which the equality is still open for $3 \le n \le 58$, namely 30, 40, 44 and 48 [LoM1].

(j) $R(K_{2,n-s}, K_{2,n}) \le 4n-2s-3$ for $s \ge 2$ and $n \ge s+2$, with the equality in many cases involving Hadamard matrices or strongly regular graphs. Asymptotics of $R(K_{2,n}, K_{2,m})$ for $m \gg n$ [LoM3].

(k) Some algebraic lower and upper bounds on $R(K_{s,n}, K_{t,m})$ for various combinations of n, m and $1 \le t, s \le 3$ [BaiLi, BaLX]. A general lower bound $R(K_{m,n}) \ge 2^m(n - n^{0.525})$ for large n [Dong].

(l) Upper bounds for $R(K_{2,2}, K_{m,n})$ for $m, n \ge 2$, with several cases identified for which the equality holds. Special focus on the cases for $m = 2$ [HaMe4].

(m) Let G be any isolate-free graph with p vertices and $q \ge 2$ edges. Then it holds that $R(K_{2,2}, G) \le 2q+1$, with the equality for $G = qK_2$ or $G = K_3$, and $R(K_{2,2}, G) \le 2p+q-2$. Some generalizations to $R(K_{2,k}, G)$ [JRB].

(n) Bounds for the numbers of the form $R(K_{k,n}, K_{k,m})$, specially for fixed k and close to the diagonal cases. Asymptotics of $R(K_{3,n}, K_{3,m})$ for $m \gg n$ [LoM2]. Asymptotic upper bounds on $R(K_{2,n}, K_{2,m})$ and on $R(K_{3,3}, K_{m,n})$ for $m \le 3$ [WaLL].

(o) $R(nK_{1,3}, mK_{1,3}) = 4n+m-1$ for $n \ge m \ge 1$, $n \ge 2$ [BES].

(p) Asymptotics for $K_{2,m}$ versus K_n [CaLRZ]. Upper bound asymptotics for $K_{k,m}$ versus K_n [LiZa1] and for some bipartite graphs K_n [JiSa].

(q) Special two-color cases apply in the study of asymptotics for multicolor Ramsey numbers for complete bipartite graphs [ChGra1].

4. Two Colors: Numbers Involving Cycles

4.1. Cycles, cycles versus paths and stars

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

Cycles

(a) $R(C_3, C_3) = 6$ [BaLiu, Bush, GG],
$R(C_4, C_4) = 6$ [ChH1].

(b) $R(C_3, C_n) = 2n-1$ for $n \geq 4$, $R(C_4, C_n) = n+1$ for $n \geq 6$,
$R(C_5, C_n) = 2n-1$ for $n \geq 5$, and $R(C_6, C_6) = 8$ [ChaS].

(c) Result obtained independently in [Ros1] and [FS1], a new simpler proof in [KáRos]:

$$R(C_m, C_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd},\ (m,n) \neq (3,3), \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ and } n \text{ even},\ (m,n) \neq (4,4), \\ \max\{n-1+m/2,\ 2m-1\} & \text{for } 4 \leq m < n,\ m \text{ even and } n \text{ odd.} \end{cases}$$

(d) $R(mC_3, nC_3) = 3n + 2m$ for $n \geq m \geq 1$, $n \geq 2$ [BES].

(e) $R(mC_4, nC_4) = 2n + 4m - 1$ for $m \geq n \geq 1$, $(n, m) \neq (1,1)$ [MiSa, LiWa1].

(f) Formulas for $R(mC_4, nC_5)$ [LiWa2].

(g) Formulas and bounds for $R(mC_n, sC_t)$ [Den2, Biel1].

(h) Characterization of all graphs critical for $R(C_4, C_n)$ [WuSR].

(i) Study of $R(S_1, S_2)$, where S_1 and S_2 are sets of cycles [Hans].
A conjecture generalizing 4.1.c stated in [Hans] was proved in [WaCh2].

(j) Unions of cycles, formulas and bounds for various cases including diagonal, different lengths, different multiplicities [MiSa, Den2], disjoint cycles versus K_n [Fuj2], and their relation to 2-local Ramsey numbers [Biel1].

(k) Asymptotics for powers of cycles [AllBS]. Exact values for squares of paths P_n and cycles C_{3n} for sufficiently large n, in particular we have $R(C_{3n}^2, C_{3n}^2) = 9n-3$ for large n [AllMRS].

Cycles versus paths

Result obtained by Faudree, Lawrence, Parsons and Schelp in 1974 [FLPS]:

$$R(C_m, P_n) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd}, \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ even}, \\ \max\{m-1+\lfloor n/2 \rfloor,\ 2n-1\} & \text{for } 2 \leq n \leq m,\ m \text{ odd}, \\ m-1+\lfloor n/2 \rfloor & \text{for } 2 \leq n \leq m,\ m \text{ even.} \end{cases}$$

For all n and m it holds that $R(P_m, P_n) \le R(C_m, P_n) \le R(C_m, C_n)$. Each of the two inequalities can become an equality, and, as derived in [FLPS], all four possible combinations of $<$ and $=$ hold for an infinite number of pairs (m, n). For example, if both m and n are even, and at least one of them is greater than 4, then $R(P_m, P_n) = R(C_m, P_n) = R(C_m, C_n)$. For related generalizations see [BEFRS2].

Cycles versus stars

Only partial results for C_m versus stars are known. Lawrence [Law1] settled the cases for odd m and for long cycles (see also [Clark, Par6]). The case for short even cycles is open, and it is related in particular to bipartite graphs. Partial results for $C_4 = K_{2,2}$ are pointed to in Sections 3.3.1 and 3.3.2. The most known general exact result [Law1] is:

$$R(C_m, K_{1,n}) = \begin{cases} 2n+1 & \text{for odd } m \le 2n+1, \\ m & \text{for } m \ge 2n. \end{cases}$$

Some new cases for even m not too small with respect to n were settled in 2016, in particular the exact values of $R(C_6, K_{1,n})$ for all $n \le 11$ were completed in [ZhaBC5]. The equality $R(C_6, K_{1,12}) = 17$ was obtained in [SunSh]. The progress on asymptotics for large even m, and exact values for large even m and n not too large were obtained in [AllŁPZ].

4.2. Cycles versus complete graphs

Since 1976, it was conjectured that $R(C_n, K_m) = (n-1)(m-1) + 1$ for all $n \ge m \ge 3$, except $n = m = 3$ [FS4, EFRS2]. Various parts of this conjecture were proved as follows: for $n \ge m^2 - 2$ [BoEr], for $n > 3 = m$ [ChaS], for $n \ge 4 = m$ [YHZ1], for $n \ge 5 = m$ [BolJY+], for $n \ge 6 = m$ [Schi1], for $n \ge m \ge 7$ with $n \ge m(m-2)$ [Schi1], for $n \ge 7 = m$ [ChenCZ1], and for $n \ge 4m+2$, $m \ge 3$ [Nik]. Open conjectured cases are marked in Table V by "conj."

In 2019, Keevash, Long and Skokan [KeeLS] proved the above conjecture for $n \ge C \log m / \log\log m$ for some absolute constant $C \ge 1$, and furthermore that for any $\varepsilon > 0$ and $n > n(\varepsilon)$, for the lower bound it holds that $R(C_n, K_m) > m \log m \gg (n-1)(m-1) + 1$ for all $3 \le n \le (1-\varepsilon) \log m / \log\log m$.

(a) The first column in Table V gives data from the first row in Table I.

(b) Joint credit [He2/JR4] in Table V refers to two cases in which Hendry [He2] announced the values without presenting the proofs, which later were given in [JR4]. The special cases of $R(C_6, K_5) = 21$ [JR2] and $R(C_7, K_5) = 25$ were solved independently in [YHZ2] and [BolJY+]. The double pointer [JaBa/ChenCZ1] refers to two independent papers, similarly as [JaAl/ZZ3], except that in the latter case [ZZ3] refers to an unpublished manuscript. For joint credits marked in Table V with "-", the first reference is for the lower bound and the second for the upper bound. The cases of the conjecture for $R(C_n, K_8)$, for $10 \le n \le 15$, are confirmed in [Ban2].

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	...	C_n for $n \geq m$
K_3	6 2.1.a	7 ChaS	9 ...	11	13	15	17	... ...	$2n-1$ ChaS
K_4	9 GG	10 ChH2	13 He4/JR4	16 JR2	19 YHZ1	22 ...	25	... ...	$3n-2$ YHZ1
K_5	14 GG	14 Clan	17 He2/JR4	21 JR2	25 YHZ2	29 BolJY+	33 ...	... ...	$4n-3$ BolJY+
K_6	18 Kéry	18 Ex2-RoJa1	21 JR5	26 Schi1	31 ...	36	41	... ...	$5n-4$ Schi1
K_7	23 Ka2-GrY	22 RaT-JR1	25 Schi2	31 CheCZN	37 CheCZN	43 JaBa/Ch+	49 Ch+	... ...	$6n-5$ Ch+
K_8	28 GR-McZ	26 RaT	29 LidP	36 ChenCX	43 ChenCZ1	50 JaAl/ZZ3	57 BatJA	... ...	$7n-6$ conj.
K_9	36 Ka2-GR	30 RaT-LaLR	33-36 LidP	41 LidP	49 BanAA	56 BanAA	65 conj.	... ...	$8n-7$ conj.
K_{10}	40-41 Ex5-Ang	36 LaLR						...	$9n-8$ conj.
K_{11}	47-50 Ex20-GoeR1	40-43 VO-BoRa						...	$10n-9$ conj.
K_{12}	53-59 Kol1-Les	43-51 BoRa						...	$11n-10$ conj.

Table V. Known Ramsey numbers $R(C_n, K_m)$;
Ch+ abbreviates ChenCZ1, for comments on joint credits see 4.2.b.

(c) Erdős et al. [EFRS2] asked what is the minimum value of $R(C_n, K_m)$ for fixed m, and they suggested that it might be possible that $R(C_n, K_m)$ first decreases monotonically, then attains a unique minimum, then increases monotonically with n. If so, then the results in [KeeLS] stated above imply that this transition of behavior happens at $n = \Theta(\log m / \log\log m)$.

(d) There exist constants $c_1, c_2 > 0$ such that $c_1(m^{3/2}/\log m) \leq R(C_4, K_m) \leq c_2(m/\log m)^2$. The lower bound, obtained by Bohman and Keevash ([BohK1] in 2010, see also 4.2.j/k below) improved over an almost 40 years old bound $c(m/\log m)^{3/2}$ by Spencer [Spe2], using the probabilistic method. The upper bound was reported in a paper by Caro, Li, Rousseau and Zhang [CaLRZ], who in turn give the credit to an unpublished work by Szemerédi from 1980. A refined upper bound, $R(C_4, K_m) \leq (1+o(1))(m/\log m)^2$, was presented by Liu and Li [LiuLi2] in 2021.

(e) Erdős, in 1981, in the Ramsey problems section of the paper [Erd3] formulated a challenge by asking for a proof of $R(C_4, K_m) < m^{2-\varepsilon}$, for some $\varepsilon > 0$. To date, no such proof is known.

(f) The lower bound asymptotics $R(C_n, K_m) = \Omega(m^{(n-1)/(n-2)}/\log m)$, for fixed n and large m, was obtained by Bohman and Keevash [BohK1]. An improvement of the latter by a polylogarithmic factor for odd n and improved exponents to $11/8$ and $11/9$ for C_5 and C_7, respectively [MuVer]. Further improvement to exponents $10/7$ and $5/4$ for C_5 and C_7, with a polylogarithmic factor, in [ConMMV]. Note that for $n = 4$ it gives the lower bound in 4.2.d above. See also [Spe2, FS4, AlRö] for previous related results.

(g) Enumeration of all (C_n, K_4)-graphs for $n \leq 7$ [JaNR].
Classification of all (C_n, K_6)-critical graphs for $n \geq 15$ [JaNS].

(h) A theta graph θ_n is obtained from the cycle C_n by adding one edge between some of its nonadjacent vertices. Summary of what is known about $R(\theta_n, K_k)$, and an additional result for $k = 6$, are collected in [BanJBJ]. The cases for $k = 5$ and $n = 7, 8, 9$ are solved in [JaBBJ], and the cases for $k = 7$ and ($n = 7$ or $n \geq 14$) are solved in [Ban1].

(i) Let $C_{\leq n}$ be the set of cycles of length at most n, and let the *girth* $g(G)$ be the length of the shortest cycle in graph G. Probabilistic lower bound asymptotics for $R(C_{\leq n}, K_m)$ [Spe2] currently is the same as for $R(C_n, K_m)$, for fixed n. However, there are clear differences already for girth 4 and 5 and small m: Backelin [Back1, Back2] found that $R(C_{\leq 4}, K_m)$ = 6, 8, 11, 15, 18 for m = 3, 4, 5, 6, 7, and that $R(C_{\leq 5}, K_m)$ = 5, 8, 10, 13, 15, also for m = 3, 4, 5, 6, 7, respectively.

(j) Erdős et al. [EFRS2] proved various facts about $R(C_{\leq n}, K_m)$, and in particular that it is equal to $2m - 1$ for $n \geq 2m - 1$, and to $2m$ for $m < n < 2m - 1$. The upper asymptotics for $R(C_{\leq n}, K_m)$ is implied in the study of independence number in graphs with odd girth n [Den1]. The following close to the diagonal exact values were obtained in [WuSL]: $R(C_{\leq n}, K_n)$ is equal to $2n$ and $2n + 1$ for odd n and even n, respectively, and $R(C_{\leq n}, K_{n+1}) = 2n + 3$ for odd $n \geq 5$ and even $n \geq 16$.

(k) $R(C_{\geq n}, K_{m_1, \ldots, m_k}) = (k-1)(n-1) + m_1$ for $m_1 \leq \cdots \leq m_k$, $5m_{k-1} + 3m_k \leq n$ [PoSu1].
The same equality holds for $R(C_n, K_{m_1, \ldots, m_k})$ for large m_i's and very large n [PoSu2].

(l) Upper bound asymptotics [BoEr, FS4, EFRS2, CaLRZ, Sud1, LiZa2, AlRö, DoLL2].

4.3. Cycles versus wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices,
while some authors use the definition $W_n = K_1 + C_n$ with $n + 1$ vertices.
The generalized wheel $W_{k,n}$ is defined by $W_{k,n} = K_k + C_n$, so $W_n = W_{1,n-1}$.
For the cases involving $W_3 = C_3$ versus C_m see Sections 3.2 and 4.2.

(a) $R(C_3, W_n) = 2n - 1$ for $n \geq 6$ [BuE3]. All critical graphs have been enumerated.
The critical graphs are unique for $n = 3, 5$, and for no other n [RaJi].

(b) $R(C_4, W_n)$ = 14, 16, 17 for n = 11, 12, 13, respectively [Tse1],
$R(C_4, W_n)$ = 18, 19, 20, 21 for n = 14, 15, 16, 17, respectively [DyDz2], and several higher values and bounds, including 9 cases of n between 18 and 44 [WuSR, WuSZR].

(c) $R(C_4, W_n) \leq n + \lceil (n-1)/3 \rceil$ for $n \geq 7$ [SuBUB], which was improved to $R(C_4, W_n) \leq n + \sqrt{n-2} + 1$ for $n \geq 11$ [DyDz2].

(d) $R(C_4, W_{q^2+1}) = q^2 + q + 1$ for prime power $q \geq 4$ [DyDz2], exact values of $R(C_4, W_{q^2+2})$ and $R(C_4, W_{q^2-i})$ for special q and small i [WuSZR].

(e) $R(C_4, W_n) = R(C_4, K_{1,n-1})$ for $n \geq 7$ [ZhaBC1, ZhaBC2]. Thus, the data in Table IVa and several items in Section 3.3.1 on $R(C_4, K_{1,n})$ apply also to wheels.

(f) Tight bounds on $R(C_4, W_n)$ for $46 \leq n \leq 93$ [NoBa].

	C_3	C_4	C_5	C_6	C_7	C_8	C_m	for
W_4	9 GG	10 ChH2	13 He4	16 JR2	19 YHZ1	22 ...	$3m-2$ ...	$m \geq 4$ YHZ1
W_5	11 Clan	9 Clan	9 He2	11 JR2	13 SuBB2	15 ...	$2m-1$ ...	$m \geq 5$ SuBB2
W_6	11 BuE3	10 JR3	13 ChvS	16 SuBB2	19 SuBB2	22 ...	$3m-2$ ...	$m \geq 4$ SuBB2
W_7	13 BuE3	9 Tse1	13 LuLL	11 LuLL	13 LuLL		$2m-1$ ...	$m \geq 10$ Ch1
W_8	15 BuE3	11 Tse1	15 LuLL	16 LuLL	19 Ch2	22 ...	$3m-2$ ...	$m \geq 6$ Ch2
W_9	17 BuE3	12 Tse1	17 LuLL	13 LuLL	17 LuLL		$2m-1$	$m \geq 13$ Ch1
W_{10}	19 BuE3	13 Tse1		16 Z1	19 Z2		$3m-2$	$m \geq 9$ Ch2
W_n for	... $2n-1$ $n \geq 6$ BuE3		$2n-1$ $n \geq 19$ Zhou2		$2n-1$ $n \geq 29$ Zhou2		large wheels	cycles

Table VI. Ramsey numbers $R(W_n, C_m)$ for $n \leq 10$, $m \leq 8$;
Ch1, Ch2, Z1, Z2 abbreviate ChenCMN, ChenCNZ, ZhaBC5, ZhaZZ, respectively.

(g) $R(C_7, W_n) = 2n-1$ for $n = 9, 10, 11$ [ZhaZZ].

(h) $R(W_n, C_m) = 2n-1$ for odd m with $n \geq 5m-6$ [Zhou2]. The range of n was extended in [ZhaZC].

(i) $R(W_n, C_m) = 3m-2$ for even $n \geq 4$ with $m \geq n-1$, $m \neq 3$, was conjectured by Surahmat et al. [SuBT1, SuBT2, Sur]. Parts of this conjecture were proved in [SuBT1, ZhaCC1, Shi5, ZhaBC2, ZhaZC], and the proof was completed in [ChenCNZ].

(j) Conjecture that $R(W_n, C_m) = 2m-1$ for odd $n \geq 3$ and all $m \geq 5$ with $m > n$ [Sur]. It was proved for $2m \geq 5n-7$ [SuBT1], and improved to $2m \geq 3n-1$ in [ChenCMN]. For further progress see also [Shi5, ZhaBC2,Sanh, RaeZ, Alw].

(k) In Table VI, observe four distinct situations with respect to the parity of m and n.

(l) Cycles are Ramsey unsaturated for some wheels [AliSur], see also comments on [BaLS] in item 5.16.e.

(m) Study of cycles versus generalized wheels $W_{k,n}$ [Sur, SuBTB, Shi5, ZhaBC2, BieDa].

4.4. Cycles versus books

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_m	for
B_2	7 RoS1	7 F*al*6	9 C*al*	11 F*al*8	13 ...	15	17	19	21	$2m-1$ ...	$m \geq 4$ F*al*8
B_3	9 RoS1	9 F*al*6	10 F*al*8	11 JR2	13 Shi5	15 F*al*8	17 ...	19	21	$2m-1$ ...	$m \geq 6$ F*al*8
B_4	11 RoS1	11 F*al*6	11 F*al*8	12 S*al*1	13 S*al*1	15 Shi5	17 Shi5	19 F*al*8	21 ...	$2m-1$ ...	$m \geq 7$ F*al*8
B_5	13 RoS1	12 F*al*6	13 F*al*8	14 S*al*1	15 S*al*1	15 S*al*2	17 S*al*2	19 Shi5	21 Shi5	$2m-1$ ...	$m \geq 8$ F*al*8
B_6	15 RoS1	13 F*al*6	15 F*al*8	16 S*al*2	17 S*al*2	18 S*al*2	18 S*al*2		21 Shi5	$2m-1$ ...	$m \geq 11$ Shi5
B_7	17 RoS1	16 F*al*6	17 F*al*8	16 S*al*2	19 S*al*2	20 S*al*2	21 S*al*2			$2m-1$	$m \geq 13$ Shi5
B_8	19 RoS1	17 Tse1	19 F*al*8	17 S*al*2	19 S*al*2	22 S*al*2	≥ 23 S*al*2			$2m-1$	$m \geq 14$ Shi5
B_9	21 RoS1	18 Tse1	21 F*al*8	18 S*al*2			≥ 25 S*al*2	≥ 26 S*al*2		$2m-1$	$m \geq 16$ Shi5
B_{10}	23 RoS1	19 Tse1	23 F*al*8	19 S*al*2				≥ 28 S*al*2		$2m-1$	$m \geq 17$ Shi5
B_{11}	25 RoS1	20 Tse1	25 F*al*8							$2m-1$	$m \geq 19$ Shi5
B_n for	... $2n+3$ $n \geq 2$ RoS1	$\approx n$ some (c)	... $2n+3$ $n \geq 4$ F*al*8		$2n+3$ $n \geq 15$ F*al*8		$2n+3$ $n \geq 23$ F*al*8		$2n+3$ $n \geq 31$ F*al*8	large books	cycles

Table VII. Ramsey numbers $R(B_n, C_m)$ for $n, m \leq 11$;
et al. abbreviations: F*al*/FRS, C*al*/ChRSPS, S*al*1/ShaXBP, S*al*2/ShaXB.

(a) For the cases of $B_1 = K_3$ versus C_m see Section 4.2.
The exact values for the cases $(3,7)$, $(4,8)$, $(4,9)$, $(5,10)$, $(5,11)$ were obtained independently in [S*al*1, S*al*2]/[ShaXBP, ShaXB] using computer algorithms.

(b) $R(C_4, B_{12}) = 21$ [Tse1], $R(C_4, B_{13}) = 22$, $R(C_4, B_{14}) = 24$ [Tse2].
$R(C_4, B_8) = 17$ [Tse2] (it was reported incorrectly in [FRS7] to be 16).
$R(C_4, B_{17}) \leq 28$ was obtained in [BoRa].

(c) $q^2 + q + 2 \leq R(C_4, B_{q^2 - q + 1}) \leq q^2 + q + 4$ for prime power q [FRS7]. B_n is a subgraph of B_{n+1}, hence likely $R(C_4, B_n) = n + O(\sqrt{n})$ (compare to $R(C_4, K_{2,n})$ in Section 3.3).

(d) $R(C_4, B_{(m-1)^2+t-2}) \leq m^2 + t$ for $m \geq 4$ and $0 \leq t \leq m-1$, and an infinite number of exact values of $R(C_4, B_n)$. In particular, $R(C_4, B_{q^2 - q - 2}) = q^2 + q - 1$ for all prime powers $q \geq 4$ [LiLP].

(e) Upper bound on $R(C_4, B_n)$ in terms of the upper bound on $R(C_4, K_{1,n})$ [BoRa].

(f) $R(B_n, C_m) = 2n + 3$ for odd $m \geq 5$ with $n \geq 4m - 13$, and
$R(B_n, C_m) = 2m - 1$ for $n \geq 1$, $m \geq 2n + 2$ [FRS9]. The range of m in the latter was extended to $m \geq 2n - 1 \geq 7$ in [ShaXB], and to $m > (6n+7)/4$ in [Shi5].

(g) Close to the diagonal we have $R(B_n, C_n) \geq 3n - 2$ and $R(B_{n-1}, C_n) \geq 3n - 4$ for $n \geq 3$ [ShaXB], and for all sufficiently large n it holds [LinP]:

$$R(B_n, C_m) = \begin{cases} 3m-2 & \text{for } 9n/10 \leq m \leq n, \\ 3n-2 & \text{if } m = n+1, \\ 3n & \text{for } n+2 \leq m \leq 10n/9. \end{cases}$$

(h) More theorems on $R(B_n, C_m)$ in [FRS7, FRS9, NiRo4, Zhou1].

(i) Cycles versus some generalized books $B_n^{(k)} = nK_1 + K_k$ [Shi5]. Exact asymptotics for odd cycles versus $B_n^{(k)}$ [LiuLi1], and for general cases close to the diagonal [LinP].

4.5. Cycles versus other graphs

(a) C_4 versus stars [Par3, Par4, Par5, BEFRS4, Chen, ChenJ, DyDz2, GoMC, MoCa, WuSR, WuSZR, SunSh]. For several exact results see the cases of $C_4 = K_{2,2}$ in the tables IVa/b/c, and for general results see several items in Sections 3.3.1, 3.3.2, and 4.3.e.

(b) C_4 versus unions of stars [HaABS, Has, HaJu]

(c) C_4 versus trees [EFRS4, Bu7, BEFRS4, Chen]

(d) C_4 versus all graphs on six vertices [JR3]

(e) C_4 versus various types of complete bipartite graphs, see [LiuLi2] and Section 3.3.

(f) $R(C_4, G) \leq 2q + 1$ for any isolate-free graph G with $q \geq 2$ edges, and the equality holds for $G = qK_2$ or $G = K_3$ [RoJa2, JRB].

(g) $R(C_4, G) \leq 2p + q - 2$ for any isolate-free graph G on p vertices and $q \geq 2$ edges [JRB].

(h) $R(C_5, K_4 - e) = 9$ [ChRSPS]

(i) $R(C_5, K_6 - e) = 17$ [JR4]

(j) C_5 versus all graphs on six vertices [JR4]

(k) $R(C_6, K_5 - e) = 17$ [JR2]

(l) C_6 versus all stars up to $K_{1,12}$ [ZhaBC5, SunSh]
C_6 versus all graphs on five vertices [JR2]

(m) $R(C_{2m+1}, G) = 2n - 1$ for sufficiently large sparse graphs G on n vertices, in particular $R(C_{2m+1}, T_n) = 2n - 1$ for all $n > 1512m + 756$, for n-vertex trees T_n [BEFRS2]. The range of n for trees in the latter was extended to $n \geq 25(2m+1)$ in [Bren2]. More on cycles versus trees [FSS2].

(n) $R(C_n, G) \leq 2q + \lfloor n/2 \rfloor - 1$, for $3 \leq n \leq 5$, for any isolate-free graph G with $q > 3$ edges. It is conjectured that it also holds for other n [RoJa2].

(o) Study of conditions for graphs G for which $R(C_n, G) = (n-1)(\chi(G) - 1) + \sigma(G)$, where $\sigma(G)$ is chromatic surplus of G [AllBS, HasHKL].

(p) Cycles versus fans and other graphs [Shi5], large even cycles versus fans [YouLin1].

(q) Exact asymptotics of odd cycles versus generalized fans [LiuLi1],
fans versus generalized books $K_k + nK_1$ [LiuLi3]

(r) Cycles versus nW_4 [Sudar4]

(s) Monotone paths and cycles [Lef]. This 1993 work was followed by many papers on the so-called *ordered Ramsey numbers*, about which only a couple of hints are made in this survey, and they are not covered here otherwise.

(t) Cycles versus $K_{n,m}$ and multipartite complete graphs [BoEr, PoSu1, PoSu2]

(u) Cycles versus generalized books and wheels [Shi5, Sur, SuBTB]

(v) Cycles versus special graphs of the form $K_n + G$ with small $n \leq 3$ and sparse G [Shi5]

5. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results, or references to them, but only those out of general bounds and other results which, in our opinion, may have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in Section 8, or be a special case of a general result listed in this section. Note that $P_2 = K_2$, $B_1 = F_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

5.1. Paths

$R(P_m, P_n) = n + \lfloor m/2 \rfloor - 1$ for all $n \geq m \geq 2$ [GeGy]

Complete classification of $R(P_m, P_n)$-critical graphs [Hook]

Stripes mP_2 [CocL1, CocL2, Lor]

Trails (paths with repeated vertices) [Osu, ConT]

Disjoint unions of paths (also called linear forests) [BuRo2, FS2]

Monotone paths [CaYZ], ordered path powers [Mub2], see also 4.5.s

Asymptotics for powers of paths [AllBS]. Exact values for squares of paths P_n for sufficiently large n, in particular we have $R(P^2_{3n}, P^2_{3n}) = R(P^2_{3n+1}, P^2_{3n+1}) = 9n - 3$, and $R(P^2_{3n+2}, P^2_{3n+2}) = 9n + 1$ for large n [AllMRS].

5.2. Wheels

Note: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices, while some authors use the definition $W_n = K_1 + C_n$ with $n + 1$ vertices.

(a) $R(W_3, W_n) = 2n - 1$ for all $n \geq 6$ [BuE3],
All critical colorings for $R(W_3, W_n)$ for all $n \geq 3$ [RaJi].

(b) The graph $3K_{m-1}$ is a witness of $3m - 2 \leq R(W_m, W_n)$ for all even n, and the graph $2K_{m-1}$ is a witness of $2m - 1 \leq R(W_m, W_n)$ for all m and n. In Table VIII, the lower bounds without a credit are implied by these inequalities.

(c) $R(W_n, W_n) \leq 8n - 10$ for even n, and $R(W_n, W_n) \leq 6n - 8$ for odd n [MaoWMS].

(d) All critical colorings (2, 1 and 2) for $R(W_n, W_6)$, for $n = 4, 5, 6$ [FM].

(e) $R(W_6, W_6) = 17$, $R(4,4) = 18$ and $\chi(W_6) = 4$ give a counterexample $G = W_6$ to the Erdős conjecture (Erd2, see also [GRS]) that $R(G, G) \geq R(K_{\chi(G)}, K_{\chi(G)})$.

(f) The value $R(W_5, W_5) = 15$ was given in the Hendry's table [He2] without a proof.
Later the proof was published in [HaMe2].

m \ n	3	4	5	6	7	8	9	10
3	6	9 GG	11 Clan	11 BuE3	13 BuE3	15 BuE3	17 BuE3	19 BuE3
4		18 GG	17 He3	19 FM	21 VO-LidP	22-26 LidP	≥ 25	
5			15 He2	17 FM	15 We-VO1	17 VO-LidP	18 VO1	≥ 21 VO
6				17 FM	19 LidP	22-26 LidP	≥ 25	
7					19 VO-LidP	19-21 LidP	≥ 19 VO	≥ 21 VO
8						22-25 LidP	≥ 25	
9							≥ 21 VO	
10								≥ 28

Table VIII. Ramsey numbers $R(W_m, W_n)$ for $m \le n \le 10$.

5.3. Books

The book graph is defined by $B_n = K_2 + nK_1$.
The generalized book $B_{k,n}$ is defined by $B_{k,n} = K_k + nK_1$, it is also written as $B_n^{(k)}$.
For cycles versus books see Section 4.4, and books versus other graphs Section 5.9.

(a) $R(B_m, B_n) \le R(B_{m+1}, B_n)$ and $R(B_m, B_n) = R(B_n, B_m)$ hold for all $m, n \ge 1$.

(b) $R(B_1, B_n) = 2n+3 \le R(B_2, B_n)$ for all $n > 1$ [RoS1].

(c) $R(B_2, B_n) \le 2n+6$ for all $n > 1$ [RoS1], $R(B_2, B_n) \le 2n+5$ for $12 \le n \le 22$, $R(B_2, B_n) \le 2n+4$ for $23 \le n \le 37$, $R(B_2, B_n) = 2n+3$ for $n \ge 38$ [FRS8].

(d) There are 4 Ramsey-critical graphs for $R(B_2, B_3)$, a unique graph for $R(B_3, B_4)$ [ShaXBP], 3 for $R(B_2, B_6)$ and 65 for $R(B_2, B_7)$ [BlLR].

(e) Unpublished result $R(B_2, B_6) = 17$ [Rou] was confirmed in [BlLR]. $R(B_{11}, B_{12}) = 47$ and $R(B_{12}, B_{13}) = 51$ were obtained in [VO1].

(f) $R(B_n, B_n) = 4n+2$ for $4n+1$ a prime power. If $4n+1$ is not the sum of two integer squares, then $R(B_n, B_n) \le 4n+1$ [RoS1].

(g) If $2(m+n)+1 > (n-m)^2/3$, then $R(B_m, B_n) \le 2(m+n+1)$ and $R(B_{n-1}, B_n) \le 4n-1$. Furthermore, if $n = 2 \pmod 3$ then $R(B_{n-2}, B_n) \le 4n-3$ [RoS1].

m \ n	1	2	3	4	5	6	7	8	9	10	11
1	6	7 ChH2	9 Clan	11 RoS1	13 RoS1	15 RoS1	17 RoS1	19 RoS1	21 RoS1	23 RoS1	25 RoS1
2		10 ChH1	11 Clan	13 Rou	16 RoS1	17 Rou	18 BlLR	21-22 VO1-*	22-24 VO1-*	25-26 VO1-*	28 FRS8
3			14 RoS1	15 Sh1+	17 RoS1	19 We-Lid					
4				18 RoS1	19 We	22 RoS1			27-28 5.3.hg		
5					21 RoS1	23 VO1					
6						26 5.3.f	27 VO1	29 VO1			
7							30 5.3.f	31 VO1		36 5.3.hg	37-38 5.3.hg
8								33 VO1	35 VO1		
9									38 5.3.f	39 VO1	41 VO1
10										42 5.3.f	43 VO1

Table IXa. Ramsey numbers $R(B_m, B_n)$ for $m \le 10$ and $m \le n \le 11$. Upper bounds marked * follow from 5.3.b/c. Other upper bounds in VO1 follow from 5.3.f/g. See the details of 5.3.b-h below, their further use leads to other bounds not listed in the table. Sh1+ abbreviates ShaXBP.

m	n	$R(B_m B_n)$	v	k	λ	μ
11	11	46	45	22	10	11
14	17	64	63	30	13	15
23	26	100	99	48	22	24
22	37	120	119	54	21	27
29	38	136	135	64	28	32
34	37	144	143	70	33	35
47	50	196	195	96	46	48
46	58	210	209	100	45	50
56	56	226	225	112	55	56
38	82	244	243	110	37	60
62	65	256	255	126	61	63
69	71	281	280	135	70	60

Table IXb. Exact values of $R(B_m, B_n)$ from strongly regular (v, k, λ, μ)-graphs on up to 280 vertices, using 5.3.g/h [NiRo3]. It includes only the cases beyond the range of Table IXa, and excludes the cases of $m = n$ for $4n + 1$ prime power, as in 5.3.f.

(h) Strongly regular graphs often provide good lower bounds. If there exists a strongly regular graph with the parameters (v, k, λ, μ), then $R(B_{\lambda+1}, B_{v-2k+\mu-1}) \geq v+1$. The lower bounds for a number of specific larger cases, like $R(B_{62}, B_{65}) = 256$ [RoS1] or $254 \leq R(B_{37}, B_{88}) \leq 255$ [Par6], are implied by the existence of a strongly regular graph with suitable parameters. 12 exact values of $R(B_m, B_n)$, beyond Table IXa, where this lower bound meets the upper bound in 5.3.g were collected by Nikiforov and Rousseau [NiRo3], and they are presented in Table IXb. For a great collection of strongly regular graphs see the website by A. E. Brouwer [Brou].

(i) $R(B_m, B_n) = 2n+3$ for all $n \geq cm$ for some $c < 10^6$ [NiRo2, NiRo3]. A strengthening of this result implying that $R(B_{\lceil n/4 \rceil}, B_n) = (9n/4 + o(1))n$ [ChenL], and more lower and upper bounds for off-diagonal cases are in [ChenLY].

(j) $R(B_n, B_n) = (4+o(1))n$ [RoS1, NiRS].

(k) For generalized books $B_n^{(k)} = nK_1 + K_k$, Conlon proved that $R(B_n^{(k)}, B_n^{(k)}) = 2^k n + o_k(n)$ [Con4]. A simplified proof, better control of the error term, and a proof that all extremal colorings for this Ramsey problem are quasirandom are in the follow-up papers: for the diagonal case [ConFW1], and off-diagonal ConFW2]. All this more than answered some old questions by Erdős and others. The lower bound $R(B_n^{(k)}, B_n^{(k)}) \geq 8n+5$, for prime powers $4n+1$, was established in [LinLi3].

(l) Other general equalities and bounds involving $R(B_m, B_n)$ can be found in [RoS1, FRS8, Par6, NiRo2, NiRo3, NiRS, LiRZ2].

5.4. Trees and forests

In this subsection T_n and F_n denote an n-vertex tree and forest, respectively.

(a) $R(T_n, T_n) \leq 4n+1$ [ErdG]. Note that if T_n were a set of all n-vertex trees, then one might say that $R(T_n, T_n) = n$, since for every graph G at least one of G or $\overline{G}$ is connected, and thus it contains an n-vertex spanning tree.

(b) $R(T_n, T_n) \geq \lfloor (4n-1)/3 \rfloor$ [BuE2], see also Section 5.15.

(c) Conjecture that $R(T_n, T_n)$ is at most $2n-2$ for even n and $2n-3$ for odd n [BuE2]. Note that this is the same as asking if $R(T_n, T_n) \leq R(K_{1,n-1}, K_{1,n-1})$. Zhao [Zhao] proved that $R(T_n, T_n) \leq 2n-2$ and thus confirmed the conjecture for even n. Independently, Ajtai et al. [AjKSS] announced a full proof for large n. This recent progress subsumes some of the results pointed to in items (d)-(m) below.

(d) For general discussion of related problems see [Bu7, FSS2, ChGra2], in particular of the conjecture that $R(T_m, T_n) \leq n+m-2$ holds for all trees [FSS2].

(e) If $\Delta(T_m) = m-2$ and $\Delta(T_n) = n-2$ then the exact values of $R(T_m, T_n)$ are known, and they are between $n+m-5$ and $n+m-3$ depending on n and m. In particular, we have $R(T_n, T_n) = 2n-5$ for even n and $R(T_n, T_n) = 2n-4$ for odd n [GuoV].

(f) Examples of families T_m and T_n (including P_n) for which $R(T_m, T_n) = n+m-c$, $c = 3, 4, 5$ [SunZ1], extending the results in [GuoV].

(g) View the tree T as a bipartite graph with parts t_1 and t_2, $t_2 \geq t_1$, then define $b(T) = \max\{2t_1 + t_2 - 1, 2t_2 - 1\}$. Then the bound $R(T, T) \geq b(T)$ holds always, $R(T, T) = b(T)$ holds for many classes of trees [EFRS3, GeGy], and asymptotically [HaŁT], but cases for inequality have been found [GrHK].

(h) Comments in [BaLS] about some conjectures on Ramsey saturation of non-star trees, which would imply that $R(T_n, T_n) \leq 2n - 2$ holds for sufficiently large n.

(i) Formulas for $R(T_m, T_n)$ for some subcases of when T_m and T_n satisfy $\Delta(T_m) = m - 3$ and $\Delta(T_n) \geq n - 3$ [SunWW].

(j) $R(T_m, K_{1,n}) \leq m + n - 1$, with equality for $(m-1) \mid (n-1)$, and for sufficiently large n for almost all trees T_m [Bu1]. Many cases were identified for which we have $R(T_m, K_{1,n}) = m + n - 2$ [Coc, ZhZ1], see also [Bu1].

(k) $R(T_m, K_{1,n}) \leq m + n$ if T_m is not a star and $(m-1) \nmid (n-1)$, and some classes of trees and stars for which the equality holds [GuoV]. Further classes of trees and stars for which we have $R(T_m, K_{1,n}) \leq m + n - c$, and for which the equality holds with $c = 3$ or $c = 2$ [YanP].

(l) In a sequence of papers [SunZ1, SunZ2, SunW, SunWW], Zhi-Hong Sun et al. obtain several exact results for $R(S, T)$, where the trees S and T have high maximum degree $\Delta \geq n - 3$, or one of them has high maximum degree and the other is a path.

(m) Formulas for some cases of brooms [EFRS3], where broom is a star with a path attached to its center. These results were extended to all diagonal cases for brooms [YuLi]. Note that a tree T_n with $\Delta(T_n) = n - 2$ is a broom, and this case is listed in 5.4.e.

(n) $R(F_n, F_n) > n + \log_2 n - O(\log\log n)$ [BuE2], forests are tight for this bound [CsKo].

(o) Forests, linear forests (unions of paths) [BuRo2, FS3, CsKo].

(p) Extensive tables of $R(T_m, T_n)$ for $6 \leq m, n \leq 8$, for many concrete pairs of trees, which were obtained through an adiabatic quantum optimization algorithm [RanMCG].

(q) Tristars and fountains [BroNN].

(r) Paths versus trees [FSS2], see also other parts of this survey involving special graphs, in particular Sections 5.5, 5.6, 5.10, 5.12 and 5.15.

5.5. Stars, stars versus other graphs

$R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ for even n and m, and $\varepsilon = 0$ otherwise [Har1]. This is also a special case of multicolor numbers for stars 6.6.e obtained in [BuRo1].

$R(K_{1,n}, K_m) = n(m-1) + 1$ by Chvátal's theorem [Chv1].

Stars versus C_4 [Par3, Par4, Par5, BEFRS4, Chen, ChenJ], until 2002
Stars versus C_4 [GoMC, MoCa, WuSZR, ZhaBC1, ZhaCC2, ZhaCC3], since 2004
Stars versus $K_{2,n}$ [Par4, GoMC]
Stars versus $K_{n,m}$ [Stev, Par3, Par4]
See also Section 3.3

$R(K_{1,4}, B_4) = 11$ [RoS2]
$R(K_{1,4}, K_{1,2,3}) = R(K_{1,4}, K_{2,2,2}) = 11$ [GuSL]

Stars versus paths [Par2, BEFRS2]
Stars versus cycles [Law1, Clark, ZhaBC5, SunSh], see also [Par6] and Section 4.1
Stars versus $2K_2$ [MeO]
Stars versus stripes mP_2 [CocL1, CocL2, Lor]
Stars versus bistars [AlmHS]
Stars versus kipas [LiZB]

Stars versus W_5 and W_6 [SuBa1]
$nK_{1,m}$ versus W_5 [BaHA]
Stars versus W_9 [Zhang2, ZhaCZ1]
Stars versus wheels [HaBA1, ChenZZ2, Kor, LiSch, HagMa]
Stars versus books [ChRSPS, RoS2]
Stars versus fans [ZhaBC3, ZhaoW, HuP2]
Stars versus trees [Bu1, Cheng, Coc, GuoV, SunZ1, SunZ2, SunWW, ZhZ1, YanP]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Stars versus almost all connected graphs on 6 vertices [LoM7]
Values and bounds on $R(S_n, K_6 - 3K_2)$ [LoM8], see also [LoM10]

Union of two stars [Gros2]
Asymptotics for double stars* [NoSZ, FloS]
Double stars versus $K_{2,q}$ and sK_2 versus $K_s + C_n$ [SuAUB]
Odd-linked double stars [KarK]
Unions of stars versus C_4 and W_5 [HaABS, Has, HaJu]
Unions of stars versus wheels [BaHA, HaBA2, SuBAU1]

5.6. Paths versus other graphs

Note: for cycles versus P_n see Section 4.1.

P_3 versus all isolate-free graphs [ChH2]
Paths versus stars [Par2, BEFRS2]
Paths versus trees [FS4, FSS2, SunZ1, SunZ2, SunWW]
Paths versus books [RoS2]
Paths versus K_n [Par1]
Paths versus $2K_n$ [SuAM, SuAAM]
Paths versus $K_{n,m}$ [Häg]
Paths versus some balanced complete multipartite graphs [Pokr]
Paths versus W_5 and W_6 [SuBa1]
Paths versus W_7 and W_8 [Bas]

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* double star is a union of two stars with their centers joined by an edge

Paths versus wheels [BaSu, ChenZZ1, SaBr3, Zhang1]
Paths versus wheels, the last piece completed [LiNing2]
$R(P_n, mW_4) = 2n + m - 2$ [Sudar1]
Paths versus beaded wheels [AliBT2]
Paths versus sunflower graphs [AliTJ]
Paths versus powers of paths [Pokr, AllBS, AllMRS]
Paths versus fans [SaBr2]
Paths versus $K_1 + P_m$ [SaBr1, SaBr4]
Paths versus kipas [LiZBBH]
Paths versus $K_1 + F$, where F is a linear forest [LiNing1]
Paths versus Jahangir graphs [SuTo]
Paths and cycles versus trees [FSS2]
Powers of paths [AllBS, AllMRS], see also Section 5.1
Unions of paths [BuRo2]
Paths and unions of paths versus tK_n [Sudar2]
Paths and unions of paths versus Jahangir graphs [AliBas, AliBT1, AliSur]
Paths and unions of paths versus $K_{2m} - mK_2$ [AliBB]
Goodness of paths for tK_n [Sudar3]
Goodness of paths, results on graphs H for which P_n is H-good [PoSu1]
Sparse graphs versus paths and cycles [BEFRS2]
Graphs with long tails [Bu2, BuG]
Long paths versus other good graphs [PeiLi, PeiCLY]
Paths versus generalized wheels [BieDa]
Monotone paths [Lef, CaYZ, MuSuk5], and monotone cycles [Lef], see also 4.5.s

5.7. Fans, fans versus other graphs

The fan graph F_n is defined by $F_n = K_1 + nK_2$,
and generalized fan $F_{k,n}$ is defined by $F_{k,n} = K_1 + nK_k$.

$R(F_2, F_2) = 9$ [Bu4]
$R(F_3, F_3) = 14$ [ZhaoW]
$R(F_1, F_n) = R(K_3, F_n) = 4n + 1$ for $n \geq 2$, and bounds for $R(F_m, F_n)$ [LiR2, GuGS]
$R(F_2, F_n) = 4n + 1$ for $n \geq 2$, and
$R(F_m, F_n) \leq 4n + 2m$ for $n \geq m \geq 2$ [LinLi1],
$R(F_m, F_n) = 4n + 1$ for n sufficierntly larger than m [LinLD].
$9n/2 - 5 \leq R(F_n, F_n) \leq 11n/2 + 6$ for all $n \geq 1$ [ChenYZ],
upper bound growth improved to $R(F_n, F_n) \leq 31n/6 + 15$ for all $n \geq 1$ [DvoMe].
$R(K_4, F_n) = 6n + 1$ for $n \geq 3$ [SuBB3]
$R(K_5, F_n) = 8n + 1$ for $n \geq 5$ [ZhaCh]
$R(K_6, F_n) = 10n + 1$ for $n \geq 6$ [KaOS]

A conjecture that $R(K_m, F_n) = 2mn - 2n + 1$ for $n \geq m \geq 4$ [SuBB3]

Fans versus paths, formulas for a number of cases including $R(P_6, F_n)$ [SaBr2], missing case $R(P_6, F_4) = 12$ solved in [Shao].

$R(F_m, K_n) \leq (1 + o(1))n^2/\log n$ [LiR2]

Lower bounds on $R(F_2, K_n)$ from cyclic graphs for $n \leq 9$ [Shao]
Fans versus wheels [ZhaBC4, MengZZ]
Fans versus stars [ZhaoW, HuP2]
Fans versus trees and stars [ZhaBC3, Bren1]
Fans versus generalized books $K_k + nK_1$ [LiuLi3]
Fans versus cycles [Shi5]
Fans versus large even cycles [YouLin1]
Fans versus unicyclic graphs [Bren1]

$R(K_3, F_{3,n}) = 6n + 1$, for $n \geq 3$ [HaoLin]
$R(K_3, F_{4,n}) = 8n + 1$, for $n \geq 4$ [WaQi]

K_m and other graphs versus generalized fans $F_{k,n}$ [LiR2, WaQi]
nK_m versus $F_{G,n} = K_1 + nG$, where $n \geq 2$ and G is a graph [HaHT]
Asymptotics of odd cycles versus generalized fans $F_{G,n}$ [LiuLi1]
Generalized fans versus special chromatic graphs G [JiHou]

5.8. Wheels versus other graphs

Notes: In this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices, while some authors use the definition $W_n = K_1 + C_n$ with $n + 1$ vertices. For cycles versus W_n see Section 4.3. Consider also similarity of wheels to other graphs, like fans, kipas [LiZBBH], sunflower [AliTJ], and Jahangir graphs [SuTo].

$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(W_5, K_5) = 27$ [He2][RaST]

$33 \leq R(W_5, K_6) \leq 36$ [ShaoWX, LidP]
$45 \leq R(W_5, K_7) \leq 50$ [VO, LidP]
$34 \leq R(W_6, K_6) \leq 40$ [VO, LidP]
$45 \leq R(W_6, K_7) \leq 55$ [GoeVO, LidP]

W_5 and W_6 versus stars and paths [SuBa1]
W_5 versus $nK_{1,m}$ [BaHA]
W_5 versus unions of stars [Has]
W_5 versus theta graphs θ_n [JaBVR]
W_5 and W_6 versus trees [BaSNM]
W_7 and W_8 versus paths [Bas]

W_7 versus trees T_n with $\Delta(T_n) \geq n - 3$, other special trees T, and T_n for $n \leq 8$ [ChenZZ3, ChenZZ5, ChenZZ6]

W_7 and W_8 versus trees [ChenZZ4, ChenZZ5]
W_9 versus stars [Zhang2, ZhaCZ1, ZhaCC4, ZhaCC7]
W_9 versus trees of high maximum degree [ZhaCZ2]
W_{2n} versus trees of high maximum degree [HafBa]
$R(C_4, W_n) = R(C_4, K_{1,n-1})$ for $n \geq 7$ [ZhaBC1].

Wheels versus stars [HaBA1, ChenZZ2, Kor, LiSch, HagMa]
Wheels W_n, for even n, versus star-like trees [SuBB1]
Wheels versus paths [BaSu, ChenZZ1, SaBr3, Zhang1]
Wheels versus paths, the last piece completed [LiNing2]
Wheels versus fans and wheels [ZhaBC4, MengZZ]
Wheels versus some trees [RaeZ, ZhuZL]
Wheels versus books [Zhou3]
Wheels versus unions of stars [BaHA, HaBA2, SuBAU1]
Wheels versus linear forests (disjoint unions of paths) [SuBa2]
Some cases of wheels versus $K_n - K_{1,s}$ [ChaMR]
Generalized wheels versus cycles [Shi5, BieDa]
Generalized wheels $W_{k,6}$ and $W_{k,7}$ versus trees [Wang2, ChngTW]
Generalized wheels versus trees [WaCh]
Upper asymptotics for $R(W_n, K_m)$ [Song5, SonBL]
Upper asymptotics for generalized wheels versus K_n [Song9]

5.9. Books versus other graphs

The book graph is defined by $B_n = K_2 + nK_1$.
The generalized book $B_{k,n}$ is defined by $B_{k,n} = K_k + nK_1$, it is also written as $B_n^{(k)}$.
For cycles versus books see also Section 4.4.

$R(B_3, K_4) = 14$ [He3]
$R(B_3, K_5) = 20$ [He2][BaRT]
$R(B_4, K_{1,4}) = 11$ [RoS2]
Cyclic lower bounds for $R(B_m, K_n)$ for $m \leq 7$, $n \leq 9$
and for $R(B_3, K_n - e)$ for $n \leq 7$ [Shao, ShaoWX]
$R(T_n, B_m) = 2n - 1$ for all $n \geq 3m - 3$ [EFRS7]
B_4 versus trees [LoM9]
Trees T_n are $2B_2$-good for $n \geq 5$ [GuoHP]
Books versus paths [RoS2]
Books versus stars [ChRSPS, RoS2]
Books versus trees [EFRS7, ZhaCZ]
Books versus K_n [LiR1, Sud2]
Books versus wheels [Zhou3]
Books versus $K_2 + C_n$ [Zhou3]
Books and $(K_1 + tree)$ versus K_n [LiR1]

A 2023 paper *Ramsey Goodness of Books Revisited* [FoxHW] reviews the subject of the title and proves a result that $R(G, B_{k,n}) = (p-1)(n-1)+1$ for generalized books $B_{k,n}$ (also called k-books), where G is a complete p-partite graph with some constraints on the sizes of its parts. A conjecture on near Ramsey goodness posed in [FoxHW] is disproved in [FanLin].

$B_{3,n}$ versus cycles [Shi5]
$B_{k,n}$ versus K_m [NiRo1, NiRo4]
$B_{k,n}$ versus $K_1 + C_4$ [LinLiu]
$B_{k,n}$ versus fans [LiuLi3]
$B_{k,n}$ versus graphs with small bandwidth [YouLC]
$C_{(2m+1)}$-goodness of $B_{k,n}$ and $K_k + nH$ [LiuLi4]

5.10. Trees and forests versus other graphs

In this subsection T_n and F_n denote n-vertex tree and forest, respectively.

$R(T_n, K_m) = (n-1)(m-1)+1$ [Chv1]

$R(C_{2m+1}, T_n) = 2n-1$ for all $n > 1512m+756$, for n-vertex trees T_n [BEFRS2].
The range of n was extended to $n \geq 25(2m+1)$ in [Bren2].

$R(T_n, B_m) = 2n-1$ for all $n \geq 3m-3$ [EFRS7]

$R(F_{nk}, K_m) = (n-1)(m-2)+nk$ for all forests F_{nk} consisting of k trees with n vertices each, also exact formula for all other cases of forests versus K_m [Stahl]

Exact results for almost all small $(n(G) \leq 5)$ connected graphs G versus all trees [FRS4]

Stripes versus trees and unicyclic graphs [HuP2]

Large trees are tK_m−good for t, $m \geq 3$ [LuoP2]

Trees versus stars [Bu1, Cheng, Coc, GuoV, ZhZ1, YanP]
Trees versus unions of complete graphs of different sizes [HuLuo]
Trees versus paths [FS4, FSS2]
Trees versus C_4 [EFRS4, Bu7, BEFRSS5, Chen]
Trees versus cycles [FSS2, EFRS6]
Trees versus books [EFRS7, ZhaCZ]
Trees versus B_4 [LoM9]
Trees T_n are $2B_2$-good for $n \geq 5$ [GuoHP]
Trees versus fans [ZhaBC3]
Trees versus W_5 and W_6 [BaSNM]
Trees versus W_7 and W_8 [ChenZZ4, ChenZZ5]
Some trees versus wheels [RaeZ, ZhuZL]
Trees versus wheels [ZhaBC4]

Trees T_n with $\Delta(T_n) \geq n-3$, other special trees T,
and T_n for $n \leq 8$ versus W_7 [ChenZZ3, ChenZZ5, ChenZZ6]

Trees T_n with $\Delta(T_n) \geq n-4$ versus W_9 [ZhaCZ2]

Trees T_n with large $\Delta(T_n)$ versus W_{2m} [HafBa]

Star-like trees versus odd wheels [SuBB1, ChenZZ3]
Trees versus $K_n + \overline{K}_m$ [RoS2, FSR]
Trees versus generalized wheels $W_{k,6}$ and $W_{k,7}$ [Wang2, ChngTW]
Trees versus generalized wheels [WaCh]
Trees versus bipartite graphs [BEFRS4, EFRS6]
Trees versus almost complete graphs [GoJa2]
Trees versus multipartite complete graphs [EFRS8, BEFRSGJ]

$R(T, G)$ for most non-star trees T and $n(G) \le 6$ [LoM10], see item 8.1.q

Linear forests versus $3K_3$ and $2K_4$ [SuBAU2]
Linear forests versus $2K_m$ [SuAAM]
Linear forests versus tK_n [Sudar2, Sudar3]
Linear forests versus wheels [SuBa2]
Forests versus almost complete graphs [ChGP]
Forests versus complete graphs [BuE1, Stahl, BaHA]
Forests versus disjoint union of complete graphs [HuP1]

Goodness of bounded degree trees [BalPS]

Study of graphs G for which all or almost all trees are G-good [BuF, BEFRSGJ], see also Section 5.15 and 5.16, item [Bu2], for the definition and more pointers.

See also various parts of this survey for special trees, and Section 5.4.

5.11. Cases for $n(G)$, $n(H) \le 5$

Clancy [Clan], in 1977, presented a table of $R(G, H)$ for all isolate-free graphs G with $n(G) = 5$ and H with $n(H) \le 4$, except 5 entries. All five of the open entries have been solved as follows:

$R(B_3, K_4) = 14$	[He3]
$R(K_5, K_4 - e) = 16$	[BoH]
$R(W_5, K_4) = 17$	[He2]
$R(K_5 - e, K_4) = 19$	[EHM1]
$R(K_5, K_4) = R(4, 5) = 25$	[MR4]

An interesting case in [Clan] is:

$$R(K_4, K_5 - P_3) = R(K_4, K_4 + e) = R(4, 4) = 18$$

Hendry [He2], in 1989, presented a table of $R(G, H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries. Five of the open entries have been solved:

$R(K_5, K_4 + e) = R(4, 5) = 25$	[Ka1][MR4]
$R(K_5, K_5 - P_3) = 25$	[Ka1][Boza2, CalSR]
$R(K_5, B_3) = 20$	[He2][BaRT]
$R(K_5, W_5) = 27$	[He2][RaST]

$R(W_5, K_5-e) = 17$ [He2][YH]

The still open cases for K_5 versus K_5-e and K_5 are:

$30 \leq R(K_5, K_5-e) \leq 33$ [Ex6][Boza7]
$43 \leq R(K_5, K_5) \leq 48$ [Ex4][AnM1]

All critical colorings for the case $R(C_5+e, K_5) = 17$ were found by Hendry [He5].

5.12. Miscellaneous cases

$R(P, P) \geq 19$, where P is the 10-vertex Petersen graph [HaKr2]

$R(Q_3, Q_3) = 13$, where Q_3 is the 8-vertex cube graph [LidP], see also item 5.15.m

$30 \leq R(K_{2,2,2}, K_{2,2,2}) \leq 31$, where $K_{2,2,2}$ is the octahedron [HaKr2, LidP]

Unicyclic graphs [Gros1, Köh, KrRod, LowKap]
$K_{2,m}$ and C_{2m} versus K_n [CaLRZ]
$K_{2,n}$ versus any isolate-free graph [RoJa2, JRB]
Union of two stars [Gros2]
Double stars [GrHK, BahS, NoSZ, FloS]
Odd-linked double stars [KarK]
Formulas for some cases of brooms+ [EFRS3], extended to all diagonal cases [YuLi]
Graphs with bridge versus K_n [Li1]
Multipartite complete graphs [BFRS, FRS3, Stev]
Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Multipartite complete graphs versus sparse graphs [EFRS4]
Graphs with long tails [Bu2, BuG]

5.13. Multiple copies of graphs, disconnected graphs

(a) $2K_2$ versus isolate-free graphs [ChH2],
nK_2 versus isolate-free graphs [FSS1].

(b) nK_2 versus mK_2, in particular $R(nK_2, nK_2) = 3n-1$ for $n \geq 1$ [CocL1, CocL2, Lor]

(c) $R(nK_3, nK_3) = 5n$ for $n \geq 2$, $R(mK_3, nK_3) = 3m+2n$ for $m \geq n \geq 2$, and
$R(mK_n, mK_n) \leq (2n-1)m + c_n$ for $n \geq 4$ and some constant c_n [BES].

(d) Lorimer and Mullins [LorMu] proved that

$$R(nK_3, mK_4) = \begin{cases} 9 & \text{for } m = n = 1, \\ 3n+3m+1 & \text{for } n \geq m > 1, \\ 3n+5 & \text{for } n \geq 2 \text{ and } m = 1, \\ 2n+4m+1 & \text{for } m \geq n \text{ and } m \geq 2. \end{cases}$$

+ broom is a star with a path attached to its center

(e) $R(nK_4, nK_4) = 7n+4$ for large n [Bu8]

(f) Stripes mP_2 [CocL1, CocL2, Lor]

(g) $mK_{1,n}$ versus $sK_{1,t}$ [LuoP1]

(h) $R(nH) = (2k - \alpha)n + c$ holds for all sufficiently large n, where H is a k-vertex isolate-free graph with independence number $\alpha = \alpha(H)$, and $c = c(H)$ is a constant dependent on H [BES]. Now, it is known that there exists $C > 0$ such that this equality holds for all $n \geq 2^{Ck}$. Further, in the case of complete graphs we have the equality $R(nK_k) = (2k - 1)n + R(K_{k-1}) - 2$ [BucSu]. Significant improvements were obtained for sparse graphs, in particular those with bounded maximum degree [SulTr].

(i) $R(mG, nH) \leq (m-1)n(G) + (n-1)n(H) + R(G, H)$ [BES]

(j) Formulas for $R(nK_3, mG)$ for all isolate-free graphs G on 4 vertices [Zeng]

(k) Variety of results for numbers of the form $R(nG, mH)$ [Bu1, BES, HaBA2, SuBAU1, SuAUB, Sudar2, Sudar3].

(l) $R(F, G \cup H) \leq \max\{ R(F, G) + n(H),\ R(F, G) \}$ [Par6]

(m) $R(G, H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases [LoM5]. Missing cases were completed in [KroMe].

(n) $nK_{1,m}$ versus W_5 [BaHA], cycles versus nW_4 [Sudar4]

(o) Results on $R(G, G)$ for $G = P_k \cup K_{1,n}$: exact formulas for $k = 2$ and $k = 3$, and lower and upper bounds for $k \geq 4$ [ZhouLMW]. In [ZhouLMW], these results are used as tools to derive some Gallai-Ramsey numbers.

(p) Trees and forests versus $K_m \cup K_n$ [HuP1],
Trees versus unions of complete graphs of different sizes [HuLuo],
Trees T_n are $2B_2$-good for $n \geq 5$ [GuoHP],
Disjoint unions of paths (linear forests) [BuRo2, FS2],
Linear forests versus $3K_3 \cup 2K_4$ [SuBAU2]

(q) Forests versus K_n [Stahl, BaHA] and W_n [BaHA]. Generalizations to forests versus other graphs G in terms of $\chi(G)$ and the chromatic surplus of G [Biel4], and for linear forests versus $2K_n$ [SuAM].

(r) Disconnected graphs versus other graphs, $R(nH, G)$ [BuE1, GoJa1, BucSu]

(s) Let $c(nK_k)$ denote the set of connected graphs containing n vertex-disjoint K_k's. Then $R(c(nK_3), c(nK_3)) = 7n-2$ for $n \geq 2$ [GySá3], and for $n \geq R(k, k)$ and $k \geq 4$ we have $R(c(nK_k), c(nK_k)) = (k^2-k+1)n-k+1$ [Rob]. These are just samples of results on the so called connected Ramsey numbers, otherwise not covered in this survey.

(t) See Section 4.1 for cases involving unions of cycles.
See also [Bu9, BuE1, LorMu, MiSa, Den2, Biel1, Biel2]

5.14. General results for special graphs

(a) $R(K_m^p, K_n^q) = R(K_m, K_n)$ for $m, n \geq 3$, $m+n \geq 8$, $p \leq m/(n-1)$ and $q \leq n/(m-1)$, where K_s^t is a K_s with an additional vertex connected to it by t edges [BEFS]. Some applications can be found in [BlLR].

(b) $R(K_{2,k}, G) \leq kq+1$ for $k \geq 3$, for isolate-free graphs G with $q \geq 2$ edges [RoJa2, JRB].

(c) $R(W_6, W_6) = 17$ and $\chi(W_6) = 4$ [FM]. This gives a counterexample $G = W_6$ to the Erdős conjecture (see [GRS]) $R(G,G) \geq R(K_{\chi(G)}, K_{\chi(G)})$, since $R(4,4) = 18$.

(d) $R(G+K_1, H) \leq R(K_{1, R(G, H)}, H)$ [BuE1].

(e) $R(\overline{K}_2+G, \overline{K}_2+G) \leq 4R(G, \overline{K}_2+G) - 2$ [LiShen].

(f) For arbitrary fixed graphs G and H, if n is sufficiently large then we have $R(K_2+G, K_1+nH) = (k+1)mn+1$, where $k = \chi(G)$ and $m = |V(H)|$ [LiR2].

(g) Study of nK_m versus $F_{G,n} = K_1+nG$, where $n \geq 2$ and G is a graph [HaHT]. Study of $R(G+K_1, nH+K_1)$ [LinLD]. Further lower bounds based on the Paley graphs, in particular for $R(K_3+\overline{K}_n, K_3+\overline{K}_n)$ [LinLS].

(h) $R(K_{p+1}, B_q^r) = p(q+r-1)+1$ for generalized books $B_q^r = K_r + qK_1$, for sufficiently large q [NiRo1]. Formula for $R(K_1+C_4, B_q^r)$ for sufficiently large q [LinLiu].

(i) Study of the cases $R(K_m, K_n - K_{1,s})$ and $R(K_m - e, K_n - K_{1,s})$, with several exact values for special parameters [ChaMR]. This study was extended to some cases involving $R(K_m - K_3)$ [MonCR].

(j) Study of $R(T+K_1, K_n)$ for trees T [LiR1]. Asymptotic upper bounds for $R(T+K_2, K_n)$ [Song7], see also [SonGQ].

(k) Bounds on $R(H+\overline{K}_n, K_n)$ for general H [LiR3]. Also, for fixed k and m, as $n \to \infty$, $R(K_k+\overline{K}_m, K_n) \leq (m+o(1))\, n^k/(\log n)^{k-1}$ [LiRZ1].

(l) Asymptotics of $R(H+\overline{K}_n, K_n)$. In particular, the order of magnitude of $R(K_{m,n}, K_n)$ is $n^{m+1}/(\log n)^m$ [LiTZ]. Upper asymptotics for $R(K_s+K_{m,n}, K_k)$ [Song9].

(m) Study of the largest k such that if the star $K_{1,k}$ is removed from K_r, $r = R(G, H)$, any edge 2-coloring of the remaining part still contains monochromatic G or H, as for K_r, for various special G and H [HoIs].

(n) Let G'' be a graph obtained from G by deleting two vertices with adjacent edges. Then $R(G, H) \leq A+B+2+2\sqrt{(A^2+AB+B^2)/3}$, where $A = R(G'', H)$ and $B = R(G, H'')$ [LiRZ2].

5.15. General results for sparse graphs

(a) $R(K_n, T_m) = (n-1)(m-1)+1$ for any tree T_m on m vertices [Chv1].

(b) Graphs yielding $R(K_n, G) = (n-1)(n(G)-1)+1$, called Ramsey n-good [BuE3], and related results [EFRS5]. An extensive survey and further study of n-goodness appeared in [NiRo4], 2009. More results on goodness of bounded degree trees [BalPS], 2016, and paths [PoSu1], 2017.

(c) $R(C_{2m+1}, G) = 2n-1$ for sufficiently large sparse graphs G on n vertices, little more complicated formulas for P_{2m+1} instead of C_{2m+1} [BEFRS2].

(d) Conjecture that $R(G,G) \le 12n(G)$ for all planar G, for sufficiently large n [AllBS].

(e) $R(G,G) \le c_d n(G)$ for all G, where constant c_d depends only on the maximum degree d in G [ChRST]. The constant was improved in [GRR1, FoxSu1]. Tight lower and upper bounds for bipartite G [GRR2, Con2, ConFS7, ConFS8]. Further improvements of the constant c_d in general were obtained in [ConFS4], and for graphs with bounded bandwidth in [AllBS].

(f) $R(G,G) \le c_d n$ for all d-arrangeable graphs G on n vertices, in particular with the same constant for all planar graphs [ChenS]. The constant c_d was improved in [Eaton]. An extension to graphs not containing a subdivision of K_d [RöTh].

(g) Ramsey numbers grow linearly for degenerate graphs versus some sparser graphs, arrangeable graphs, crowns, graphs with bounded maximum degree, planar graphs, and graphs without any topological minor of a fixed clique [Shi3].

(h) Study of L-sets, which are sets of pairs of graphs whose Ramsey numbers are linear in the number of vertices. Conjecture that Ramsey numbers grow linearly for d-degenerate graphs (graph is d-degenerate if all its subgraphs have minimum degree at most d) [BuE1], 1973. Progress towards this conjecture was obtained by several authors, including [KoRö1, KoRö2, KoSu, FoxSu1, FoxSu2]. Further progress, and the proof of the conjecture were obtained in 2017, using its relation to the chromatic number [Lee].

(i) Ramsey number is *linear* in a class of graphs X if $R_X(p, q) \le c(p+q)$ for some constant c and all p, q, where we color the edges of graphs in X. A conjecture that this linearity holds for X if and only if the co-chromatic number is bounded in X [AtLZ]. Discussion of various old and new classes of Ramsey linear graphs [NeOs]. A similar concept of classes of graphs for other constraints on $R(p, q)$ are studied in [Loz].

(j) Study of graphs G, called *Ramsey size linear,* for which there exists a constant c_G such that for all H with no isolates $R(G, H) \le c_G e(H)$ [EFRS9], 1993. An overview and further results were given in [BaSS], 2002, and more recently in [BraGS], 2024.

(k) $R(G, G) < 6n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent [LiRS]. This improves the result $R(G, G) \le 12n$ in [Alon1]. In an early paper by Burr and Erdős [BuE1] it was proved that if any two points of degree at least 3 are at distance at least 3 then $R(G, G) \le 18n$.

(l) $R(G_{a,b}, G_{a,b}) = (3/2 + o(1))ab$, where $G_{a,b}$ is the rectangular $a \times b$ grid graph. Other similar results follow for bipartite planar graphs with bounded degree and grids of higher dimension [MoSST].

(m) $R(Q_n, Q_n) \le 2^{2.62n + o(n)}$, for the n-dimensional hypercube Q_n with 2^n vertices [Shi1]. This bound can also be derived from a theorem in [KoRö1]. An improvement was obtained in [Shi4], a further one to $R(Q_n, Q_n) \le 2^{2n+5}n$ in [FoxSu1], then another decrease of the upper bound to 2^{2n+6} in [ConFS8], and the latest one to $2^{2n-cn+1}+2$, for some positive c and sufficiently large n [Tikh].
A lower bound construction for $12 \le R(Q_3, Q_3)$ was presented in [HaKr2], and

$R(Q_3, Q_3) = 13$ was shown in [LidP]. This is not be be confused with the poset Ramsey numbers for Q_n (studied in [BohP], for example, but otherwise not covered in this survey), where the vertices of Q_m are colored and monochromatic copies of Q_n are being avoided.

(n) $R(K_m, Q_n) = (m-1)(2^n - 1) + 1$ for every fixed m and sufficiently large n [FizGMSS]. This improves on the results in [ConFLS] and [GrMFSS]. The apparent contradiction with publication years is due to the timing of publication processes.

(o) Conjecture that $R(G, G) = 2n(G) - 1$ if G is unicyclic of odd girth [Gros1]. Further support for the conjecture was given in [Köh, KrRod, LowKap].

(p) Using SAT to compute $R(G, G)$ for several small sparse graphs G [LowKKB].

(q) See also earlier subsections 5.* for various specific sparse graphs.

5.16. General results

(a) $R(G, H) \geq (\chi(G) - 1)(c(H) - 1) + 1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H. [ChH2].

(b) $R(G, G) > (s\, 2^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G. Hence $R(K_{n,n}, K_{n,n}) > 2^n$, see also item 6.7.l [ChH3].

(c) $R(G, G) \geq \lfloor (4n(G) - 1)/3 \rfloor$ for any connected G, and $R(G, G) \geq 2n - 1$ for any connected nonbipartite G. These bounds can be achieved for all $n \geq 4$ [BuE2].

(d) Graphs H yielding $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$, where $s(G)$ is the chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good) [Bu2]. This idea is a basis of a number of exact results for $R(G, H)$ for large and sparse graphs H [BuG, BEFRS2, BEFRS3, Bu5, FaSi, EFRS4, FRS3, BEFSRGJ, BuF, LiR4, Biel2, SuBAU3, Song6, AllBS, PeiLi, PeiCLY, LiBie, BalPS, PoSu1, PoSu2, LinLiu]. Surveys of this area appeared in [FRS5, NiRo4].

(e) Graph G is Ramsey saturated if $R(G+e, G+e) > R(G, G)$ for every edge e in $\overline{G}$. The paper [BaLS] contains several theorems involving cycles, cycles with chords and trees on Ramsey saturated and unsaturated graphs, and also seven conjectures including one stating that almost all graphs are Ramsey unsaturated. Some classes of graphs were proved to be Ramsey unsaturated [Ho]. Special cases involving cycles and Jahangir graphs were studied in [AliSur]. Discussion of how much $R(G, G)$ can change if one vertex from G is removed [Wig2].

(f) Relations between $R(3, k)$ and graphs with large $\chi(G)$ [BiFJ]. Further detailed study of the relation between $R(3, k)$ and the chromatic gap [GySeT].

(g) $R(G, H) > h(G, d)\, n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d [Bra3].

(h) Lower asymptotics of $R(G, H)$ depending on the average degree of G and the size of H [DoLL1]. This continued the study initiated in [EFRS5], later much enhanced for both lower and upper bounds in [Sud3].

(i) Lower bound asymptotics of $R(G, H)$ for large dense H [LiZa1].

(j) A conjecture posed by Erdős in 1983 that there exists a constant c such that $R(G, G) \leq 2^{c\sqrt{e(G)}}$ for all isolate-free graphs G [Erd4]. Discussion of this conjecture and partial results, proof for bipartite graphs and progress in other cases are included in [AlKS]. In 2011, Sudakov [Sud4] completed the proof of this conjecture. An extension of the latter to some off-diagonal cases is presented in [MaOm1], and an improvement of the constant for bipartite graphs is given in [JoPe]. For the multicolor case see item 6.7.k.

(k) Lower bound on $R(G, K_n)$ depending on the density of subgraphs of G [Kriv]. This construction for $G = K_m$ produces a bound similar to the best known probabilistic lower bound by Spencer [Spe2]. Further lower and upper bounds on $R(G, K_n)$ in terms of n and $e(G)$ can be found in [Sud3].

(l) Upper bounds on $R(G, K_n)$ for dense graphs G [Con3].

(m) The graphs K_n and $K_n + K_{n-1}$ are Ramsey equivalent for $n \geq 4$, i.e. every graph arrows both of them or neither of them. This equivalence does not hold for $n = 3$, and every graph witnessing such nonequivalence contains K_6 [BlLi]. See references therein for history and further results on Ramsey equivalent and nonequivalent pairs of graphs.

(n) Relations between the cases of G or $G + K_1$ versus H or $H + K_1$ [BuE1].

(o) Study of cyclic graphs yielding lower bounds for Ramsey numbers. Exact formulas for paths and cycles, and values for small complete graphs and for graphs with up to five vertices [HaKr1].

(p) Relations between some Ramsey graphs and block designs [Par3, Par4].

(q) Lidický and Pfender used flag algebras to constrain the space of feasible Ramsey colorings of various types. This was implemented, and then led to a number of new upper bounds listed throughout this survey [LidP].

(r) Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers [Li2]. See also Section 6 in [Ros2], and [XuR3].

(s) Given integer m and graphs G and H, determining whether $R(G, H) \leq m$ holds is NP−hard [Bu6]. Further early complexity results related to Ramsey theory were presented by Burr in [Bu10]. Complexity classification of (P_k, P_l)-goodness was completed in [HasHR].

(t) Ramsey arrowing is Π_2^p−complete, a rare natural example of a problem higher than NP in the polynomial hierarchy of computational complexity theory [Scha]. Superpolynomial lower bound on the length of resolution proofs that a graph is Ramsey is obtained in [LauPRT].

(u) Ramsey numbers with prescribed rate of growth [PavPS].

(v) Special cases of multicolor results listed in Section 6.
See also surveys listed in Section 8.

6. Multicolor Ramsey Numbers

Until 2016, the only known value of a multicolor classical Ramsey number was:

$R_3(3) = R(3,3,3) = R(3,3,3;2) = 17$	[GG]
2 critical colorings (on 16 vertices)	[KaSt, LayMa]
2 colorings on 15 vertices	[Hein]
115 colorings on 14 vertices	[PR1]

Now, we know one more case, namely $R(3,3,4) = 30$. For some details see 6.1.c.

6.1. Bounds for classical numbers

General upper bound, implicit in [GG]:

$$R(k_1, \ldots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \ldots, k_{i-1}, k_i - 1, k_{i+1}, \ldots, k_r) \qquad \text{(a)}$$

The inequality in (a) is strict if the right hand side is even and at least one of the terms in the summation is even. It is suspected that this upper bound is never tight for $r \ge 3$ and $k_i \ge 3$, except for $r = k_1 = k_2 = k_3 = 3$. However, only two parameter cases are known to improve over (a), namely $R_4(3) \le 62$ [FeKR], and $R(3,3,4) \le 31$ [PR1, PR2], $R(3,3,4) \le 30$ [CodFIM], for which (a) produces the bounds of 66 and 34, respectively.

Diagonal Cases

r \ m	3	4	5	6	7	8	9	10
3	17 GG	128 HiIr	454 Ex23	1106 Row3	3214 XuR1	7174 Row5	15041 Row5	23094 Row5
4	51 Chu1	634 XXER	4073 Row3	23502 Row5	94874 Row5	182002 Row5	719204 Row5	
5	162 Ex10	4176 Row1	41626 Row5	258506 Row5				
6	538 FreSw	32006 Row1	441606 Row5					
7	1698 Row4	160024 Row1						
8	5288 Row3							
9	17805 AgCP+							

Table X. Known lower bounds for small parameter diagonal multicolor Ramsey numbers $R_r(m)$, with references.

A general construction of linear Ramsey graphs as described by Rowley [Row2, Row3] in 2020 leads to lower bounds in higher cases, such as $R_6(6) \geq 4515702$. The templates by Rowley were generalized, which yielded many new lower bound constructions for Schur numbers, which in turn give lower bounds for Ramsey numbers $R_k(3)$ for $10 \leq k \leq 15$ [AgCP+]. Other lower bounds, implied by general constructions such as those in Section 6.2, are not listed.

The most studied and intriguing open case is

$$\text{[Chu1]} \quad 51 \leq R_4(3) = R(3,3,3,3) \leq 62 \quad \text{[FeKR]}$$

The construction for $51 \leq R_4(3)$ as described in [Chu1] is correct, but be warned of a typo found by Christopher Frederick in 2003 (there is a triangle $(31, 7, 28)$ in color 1 in the displayed matrix). It was shown that the bound 51 cannot be improved by using group partitioning into disjoint union of symmetric product-free sets [Ana1], neither by some other group partition means [Ana2]. The inequality 6.1.a implies $R_4(3) \leq 66$, Folkman [Fol] in 1974 improved this bound to 65, and Sánchez-Flores [Sán] in 1995 proved $R_4(3) \leq 64$.

The upper bounds in $162 \leq R_5(3) \leq 307$, $538 \leq R_6(3) \leq 1838$, $1698 \leq R_7(3) \leq 12861$, $128 \leq R_3(4) \leq 230$ and $634 \leq R_4(4) \leq 6306$ are implied by 6.1.a (we repeat lower bounds from Table X just to see easily the ranges). All the latter and other upper bounds obtainable from known smaller bounds and 6.1.a can be computed with the help of a LISP program written by Kerber and Rowat [KerRo].

Off-Diagonal Cases

Three colors:

k \ m	4	5	6	7	8	9	10	11	12	13	14	15	16
3	30 Ka2	45 Ex2	61 ExT	85 Ex18	103 Ex18	129 Ex18	150 ExT	174 ExT	194 ExT	217 ExT	242 ExT	269 ExT	291 ExT
4	55 KrLR	89 Ex17	117 Ex17	152 ExT	193 6.2.g	242 ExT							
5	89 Ex17	139 Ex17	181 Ex17	241 6.2.g									

Table XI. Known nontrivial lower bounds for 3-color Ramsey numbers of the form $R(3, k, m)$, with references. See also 6.1.b/c/d below.

(b) In several past revisions of this survey we wrote: "The other most studied, and perhaps the only open case of a classical multicolor Ramsey number, for which we can anticipate exact evaluation in the not-too-distance future is

$$\text{[Ka2]} \quad 30 \leq R(3,3,4) \leq 31 \quad \text{[PR1, PR2]}$$

In [PR1] it was conjectured that $R(3,3,4) = 30$, and the results in [PR2] eliminate some cases which could give $R(3,3,4) = 31$". Since 2016, we can write that $R(3,3,4) = 30$ due to the computations completed by Codish, Frank, Itzhakov and Miller [CodFIM].

(c) In addition to Table XI, the bounds $303 \le R(3,6,6)$, $609 \le R(3,7,7)$ and $1689 \le R(3,9,9)$ were derived in [XXER] (used there for building other lower bounds for some diagonal cases). These three bounds were improved to 338, 674 and 1844, respectively, by Rowley [Row5].

(d) The upper bounds in the inequalities $45 \le R(3,3,5) \le 57$, $55 \le R(3,4,4) \le 77$ and $89 \le R(3,4,5) \le 158$ are implied by 6.1.a. We repeat lower bounds from Table XI to show explicitly the current ranges.

(e) In 2015, Exoo and Tatarevic obtained several lower bounds improvements which are marked as [ExT] in Table XI. The same paper improves also on several classical two-color cases in Table I, see also comments 2.1.n and 2.1.o.

(f) For three colors, Rowley [Row5] gives also the lower bounds:

$338 \le R(3,6,6)$	$674 \le R(3,7,7)$
$941 \le R(3,8,8)$	$1844 \le R(3,9,9)$
$2841 \le R(3,10,10)$	$10769 \le R(8,8,9)$

(g) Four colors:

$97 \le R(3,3,3,4) \le 149$	[Ex17], 6.1.a
$174 \le R(3,3,4,4) \le 450$	[Row2], 6.1.a
$381 \le R(3,4,4,4) \le 1577$	6.2.j, 6.1.a
$162 \le R(3,3,3,5)$	[XXER]
$513 \le R(3,3,3,10)$	6.2.g
$597 \le R(3,3,3,11)$	6.2.g
$729 \le R(3,4,5,5)$	[Row2]
$1430 \le R(3,5,5,5)$	[Row5]

Lower bounds for higher numbers can be obtained by using general constructive results from Section 6.2 below. For example, the bounds $261 \le R(3,3,15)$ and $247 \le R(3,3,3,7)$ were not published explicitly but are implied by 6.2.g and 6.2.h, respectively.

6.2. General results for complete graphs

(a) $R(k_1, \ldots, k_r) \le 2 - r + \sum_{i=1}^{r} R(k_1, \ldots, k_{i-1}, k_i - 1, k_{i+1}, \ldots, k_r)$ [GG].

(b) $R_r(3) \ge 3R_{r-1}(3) + R_{r-3}(3) - 3$ [Chu1].

(c) $R_r(m) \ge c_m(2m-3)^r$, and some slight improvements of this bound for small values of m were described in [AbbH, Gi1, Gi2, Song2]. For $m = 3$, the best known lower bound is $R_r(3) \ge (3.28...)^r$ [AgCP+].

(d) $R_r(3) \le r!(e - e^{-1} + 3)/2 \approx 2.67\, r!$ [Wan] improved over the classical upper bound $3r!$ in [GG, GRS]. This was further improved to $R_r(3) \le r!(e - 1/6) + 1 \approx 2.55\, r!$ for all $r \ge 4$ [XuXC]. Drawing from the latter, further conditional upper bounds depending on the value of $R_4(3)$ were obtained in [Eli]. In particular, assuming that $R_4(3) = 51$, we have $R_r(3) \le r!(e - 5/8) + 1 \approx 2.09\, r!$ for all $r \ge 4$.

(e) The limit $L = \lim_{r \to \infty} R_r(3)^{1/r}$ exists, though it can be infinite [ChGri]. It is known that $3.28... < L$, as implied by (c) above. The lower bounds on the limits $\lim_{r \to \infty} R_r(k)^{1/r}$ for small fixed k are gathered in [Row1, Row3, Row5], see also 6.2.v. The best lower bounds for $R_r(k)$ from the k-th residue Paley graphs for $k = 3$ and $k = 4$ are described in [DawMc], though they are much weaker than those in Table X. For some older related results, mostly on the asymptotics of $R_r(3)$, see [AbbH, Fre, Chu2, GRS, GrRö].

(f) In 2020, the limit $\lim_{r \to \infty} R_r(3)^{1/r}$ was studied by Fox, Pach and Suk [FoxPS1] assuming a conjecture for multicolorings with bounded VC-dimension, and further for $\lim_{r \to \infty} R_r(k)^{1/r}$ when restricted to the so-called semi-algebraic colorings [FoxPS2].

(g) $R(3, k, l) \ge 4R(k, l-1) - 3$ for $k \ge 3$, $l \ge 5$, and in general for $r \ge 2$ and $k_i \ge 2$ it holds
$R(3, k_1, ..., k_r) \ge 4R(k_1 - 1, k_2, ..., k_r) - 3$ for $k_1 \ge 5$, and
$R(k_1, 2k_2 - 1, k_3, ..., k_r) \ge 4R(k_1 - 1, k_2, ..., k_r) - 3$ for $k_1 \ge 5$ [XuX2, XXER].

(h) $R(3, 3, 3, k_1, ..., k_r) \ge 3R(3, 3, k_1, ..., k_r) + R(k_1, ..., k_r) - 3$ [Rob2].
For $r+1$ colors, avoiding K_3 in the first r colors and avoiding K_m in the last color, $R(3, ..., 3, m) \le r!\, m^{r+1}$ [Sár1].

(i) $R(k_1, ..., k_r) \ge S(k_1, ..., k_r) + 2$, where $S(k_1, ..., k_r)$ is the generalized Schur number [AbbH, Gi1, Gi2]. In particular, the special case $k_1 = ... = k_r = 3$ has been widely studied [Fre, FreSw, Ex10, Rob3, Row4, AgCP+].

(j) $R(k_1, ..., k_r) \ge L(k_1, ..., k_r) + 1$, where $L(k_1, ..., k_r)$ is the maximal order of any cyclic $(k_1, ..., k_r)$–coloring, which can be considered a special case of Schur partitions defining (symmetric) Schur numbers. Many lower bounds for Ramsey numbers were established by cyclic colorings. The following recurrence can be used to derive lower bounds for higher parameters. In 1968, Giraud [Gi2] showed that for $k_i \ge 3$ we have

$$L(k_1, ..., k_r, k_{r+1}) \ge (2k_{r+1} - 3)L(k_1, ..., k_r) - k_{r+1} + 2.$$

In 2017, this was superseded by a result obtained by Rowley [Row1]:

$$L(k_1, ..., k_{p+q}) \ge ((2L(k_1, ..., k_p) - 1)(2L(k_1, ..., k_q) - 1) + 1)/2.$$

It applies not only to cyclic but also to general linear graphs. In many cases it can be surpassed by constructions involving special templates [Row3], which in turn are further enhanced for Schur numbers [AgCP+].

(k) $R_r(m) \ge p + 1$ and $R_r(m+1) \ge r(p+1) + 1$ if there exists a K_m–free cyclotomic r–class association scheme of order p [Math].

(l) If the quadratic residues Paley graph Q_p of prime order $p = 4t + 1$ contains no K_k, then $R(s, k+1, k+1) \geq 4ps - 6p + 3$ [XXER].

(m) $R_r(pq+1) > (R_r(p+1)-1)(R_r(q+1)-1)$ [Abb1]

(n) $R_r(pq+1) > R_r(p+1)(R_r(q+1)-1)$ for $p \geq q$ [XXER]

(o) $R(p_1q_1+1, \ldots, p_rq_r+1) > (R(p_1+1, \ldots, p_r+1)-1)(R(q_1+1, \ldots, q_r+1)-1)$ [Song3]

(p) $R_{r+s}(m) > (R_r(m)-1)(R_s(m)-1)$ [Song2]

(q) $R(k_1, k_2, \ldots, k_r) > (R(k_1, \ldots, k_i) - 1)(R(k_{i+1}, \ldots, k_r) - 1)$ in [Song1], see [XXER].

(r) $R(k_1, k_2, \ldots, k_r) > (k_1 + 1)(R(k_2 - k_1 + 1, k_3, \ldots, k_r) - 1)$ [Rob4]

(s) Further lower bound constructions, though with more complicated assumptions, were presented in [XuX2, XXER].

(t) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.4.7) to more colors and to hypergraphs [Grol3] (item 7.4.n).

All lower bounds in (b) through (t) above are constructive. Item (h) generalizes (b), (o) generalizes both (m) and (q), and (q) generalizes (p). (n) is stronger than (m). Finally, we note that the construction in (o) with $q_1 = \ldots = q_i = 1 = p_{i+1} = \ldots = p_r$ is the same as (q).

(u) $R(n, n, n) \leq R(n-2, n, n) + 8R(n-1, n-1, n) - 6$ for $n \geq 3$ [HTHZ2].

(v) A conjecture that $R(k_1, k_2, \ldots, k_r) \geq R(k_1, k_2, \ldots, k_{r-2}, k_{r-1} - 1, k_r + 1)$ holds for all $k_r \geq k_{r-1} \geq 3$ (called DC), its implications, evidence for validity, and related problems [LiaRX]. For two-color case see also item 2.3.f. If we set $L_k = \lim_{r\to\infty} R_r(k)^{1/r}$, then the limit L_k exists, finite or infinite, for every $k \geq 3$ [ChGri]. If DC holds, then all L_k's are finite or all of them are infinite [LiaRX]. See also 6.2.e.

(w) In 2020, Conlon and Ferber [ConFer] showed constructively that $R_3(k) > 2^{7k/8+o(k)}$ and $R_4(k) > 2^{k/2}3^{3k/8+o(k)}$, and they discussed more general best known lower and upper bounds on $R_r(k)$. An improvement to their construction by Wigderson [Wig1] yields $R_r(k) \geq (2^{3r/8-1/4})^{k-o(k)}$, for any fixed $r \geq 2$, and a further improvement by Sawin [Saw] to 0.383796 in the exponent in place of 3/8 (= 0.375).

(x) Exact asymptotics of a very special but important case is known, namely we have $R(3, 3, n) = \Theta(n^3 \text{poly-}\log n)$ [AlRö]. Generalizations to other parameters and more colors are studied in [HeWi].

(y) Mattheus and Verstraëte [MatVer] established the lower bound, and He and Wigderson [HeWi] the upper bound, as in

$$c_1 k^5/\log^8 k \leq R(4, 4, k) \leq c_2 k^5/\log^4 k,$$

see also 6.6.o/p. Generalizations to other parameters and more colors are discussed in [HeWi]. For earlier results on general upper bounds and more asymptotics see [Chu4, ChGra2, ChGri, GRS, GrRö].

6.3. Cycles

Note: The paper *Ramsey Numbers Involving Cycles* [Ra4] is based on the revision #12 of this survey. It collects and comments on the results involving cycles versus any graphs, in two or more colors. It contains some more details than this survey, but only until 2009.

6.3.1. Three colors

(a) One long cycle.

The first larger paper in this area by Erdős, Faudree, Rousseau and Schelp [EFRS1] appeared in 1976. It gives several formulas and bounds for $R(C_m, C_n, C_k)$ and $R(C_m, C_n, C_k, C_l)$ for large m. For three colors [EFRS1] includes:

$R(C_m, C_{2p+1}, C_{2q+1}) = 4m - 3$ for $p \geq 2$, $q \geq 1$,
$R(C_m, C_{2p}, C_{2q+1}) = 2(m+p) - 3$ and
$R(C_m, C_{2p}, C_{2q}) = m + p + q - 2$ for p, $q \geq 1$ and large m.

(b) Triple even cycles.

$R_3(C_{2m}) \geq 4m$ for all $m \geq 2$ [DzNS], see also 6.3.2.d/e/f. It was proven that $R(C_n, C_n, C_n) = (2 + o(1))n$ for even n [FiŁu1, GyRSS], which was improved to exactly $2n$, for large n, by Benevides and Skokan [BenSk]. In 2005, Dzido [Dzi1] conjectured that $R_3(C_{2m}) = 4m$ for all $m \geq 3$. The first open case is for $R_3(C_{10})$, known to be at least 20. A more general result holds for some off-diagonal cases [FiŁu1]:

$$R(C_{2\lfloor \alpha_1 n \rfloor}, C_{2\lfloor \alpha_2 n \rfloor}, C_{2\lfloor \alpha_3 n \rfloor}) = (\alpha_1 + \alpha_2 + \alpha_3 + \max\{\alpha_1, \alpha_2, \alpha_3\} + o(1))n, \text{ for all } \alpha_1, \alpha_2, \alpha_3 > 0.$$

The conjectured equality $R_3(C_{2m}) = 4m$, whenever true, implies $R_3(P_{2m+1}) = 4m + 1$ [DyDR] (see also Section 6.4). For general mixed-parity case see 6.3.1.d/e below.

(c) Triple odd cycles.

Bondy and Erdős conjectured that $R(C_n, C_n, C_n) \leq 4n - 3$ for all $n \geq 4$ (see for example [Erd2]). If true, then for all odd $n \geq 5$ we have $R(C_n, C_n, C_n) = 4n - 3$. The first open case is for $R_3(C_9)$, known to be at least 33. Erdős [Erd3] and other authors credit this conjecture to Bondy and Erdős, often pointing to a 1973 paper [BoEr]. Interestingly, however, the conjecture is not mentioned in this paper.

Łuczak proved that $R(C_n, C_n, C_n) \leq (4 + o(1))n$, with equality for odd n [Łuc]. The result $R_3(C_{2m+1}) = 8m + 1$ for all sufficiently large m, or equivalently $R(C_n, C_n, C_n) = 4n - 3$ for large odd n, was announced with an outline of the proof by Kohayakawa, Simonovits and Skokan [KoSS1], followed by the full proof in [KoSS2].

$m\ n\ k$	$R(C_m, C_n, C_k)$	references	general results
3 3 3	**17**	GG	2 critical colorings [KaSt, LayMa]
3 3 4	17	ExRe	
3 3 5	21	Sun1+/Tse3	$5k-4$ for $k \geq 5$, $m=n=3$ [Sun1+]
3 3 6	26	Sun1+	
3 3 7	31	Sun1+	
3 4 4	12	Schu	
3 4 5	13	Sun1+/Rao/Tse3	
3 4 6	13	Sun1+/Tse3	
3 4 7	15	Sun1+/Tse3	
3 5 5	17	Tse3/LidP	
3 5 6	21	Sun1+	
3 5 7	25	Sun1+	
3 6 6	15-18	LidP	
3 6 7	21	Sun1+	
3 7 7			
4 4 4	**11**	BiaS	1000 critical colorings [Ra4]
4 4 5	12	Sun2+/Tse3	
4 4 6	12	Sun2+/Tse3	$k+2$ for $k \geq 11$, $m=n=4$ [Sun2+]
4 4 7	12	Sun2+/Tse3	values for $k=8$, 9, 10 are 12, 13, 13 [Sun2+]
4 5 5	13	Tse3	
4 5 6	13	Sun1+	
4 5 7	15	Sun1+	
4 6 6	11	Tse3	
4 6 7	13	Sun1+/Tse3	
4 7 7			
5 5 5	**17**	YR1	1701746176 critical colorings [Nar]
5 5 6	21	Sun1+	
5 5 7	25	Sun1+	
5 6 6	15-17	LidP	
5 6 7	21	Sun1+	
5 7 7			
6 6 6	**12**	YR2	$R_3(C_{2q}) \geq 4q$ for $q \geq 2$ [DzNS]
6 6 7	15	Sun1+	see 6.3.1.a for larger parameters
6 7 7			see 6.3.1.a for larger parameters
7 7 7	**25**	FSS3	$R_3(C_{2q+1}) = 8q+1$ for large q [KoSS1, KoSS2]
8 8 8	**16**	Sun/SunY	$R_3(C_{2q}) = 4q$ for large q [BenSk]

Table XII. Ramsey numbers $R(C_m, C_n, C_k)$ for $m, n, k \leq 7$ and $m=n=k=8$;
Sun1+ abbreviates SunYWLX, Sun2+ abbreviates SunYLZ2,
the work in [SunYWLX] and [SunYLZ2] is independent from [Tse3].

(d) Three mixed-parity cycles.

Ferguson [Ferg] shows that $R(C_m, C_n, C_k) = \max\{2m+n, 2n+m, (n+m)/2+k-2\}$, for all m, n, k sufficiently large, which generalizes and improves on all even case in [FiŁu1]. The reference [Ferg] consists of a Ph.D. thesis and three long arXiv preprints.

(e) Asymptotics for triples of cycles of mixed parity similar in form to (b) [FiŁu2].

(f) $R(C_3, C_3, C_k) = 5k-4$ for $k \geq 5$ [SunYWLX], and $R(C_4, C_4, C_k) = k+2$ for $k \geq 11$ [SunYLZ2]. All exceptions to these formulas for small k are listed in Table XII.

(g) Almost all of the off-diagonal cases in Table XII required the use of computers.

6.3.2. More colors

k \ m	3	4	5	6	7	8
3	17	11	17	12	25	16
4	51 62	18	33 77	18 20	49	20
5	162 307	27 29	65	26	97	28
6	538 1838	34 38	129		193	

Table XIII. Known values and bounds for $R_k(C_m)$ for small k, m;

(a) For the entries in the row $k = 3$ and in the column $m = 3$ in Table XIII, more details and all corresponding references are in Sections 6.3.1 and 6.1, respectively. The lower bounds for $m = 5, 7$ are implied by 6.3.2.l. The bound $R_4(C_5) \le 158$ follows from 6.3.2.k, and using a reasoning as in [Li4] and the equality $R_3(C_5) = 17$ one can obtain $R_4(C_5) \le 137$. The bound $R_4(C_5) \le 77$ was obtained in 2020 with the help of flag algebras [LidP]. The references to other cases with $k, m \ge 4$ can be found below in this section.

$R_4(C_4) = 18$	[Ex2] [SunYLZ1]
$33 \le R_4(C_5) \le 77$	[6.3.2.l] [LidP]
$18 \le R_4(C_6) \le 20$	[SunYJLS] [ZhaSW]
$27 \le R_5(C_4) \le 29$	[LaWo1]
$R_5(C_6) = 26$	[SunYJLS] [SunYW]
$34 \le R_6(C_4) \le 38$	[Ex22] [Boza8]
$24 \le R(C_3, C_4, C_4, C_4) \le 27$	[DyDz1] [XuR2]
$30 \le R(C_3, C_3, C_4, C_4) \le 36$	[DyDz1] [XuR2]
$49 \le R(C_3, C_3, C_3, C_4) \le 57$	[6.7.h] [BoRa]
$18 \le R(C_4, C_6, C_6, C_6) \le 20$	[ZhaSW]
$18 \le R(C_4, C_4, C_6, C_6) \le 20$	[ZhaSW]
$R(C_4, C_4, C_4, C_6) = 19$	[ZhaSW]

(b) $R_k(C_4) \le k^2 + k + 1$ for all $k \ge 1$, $R_k(C_4) \ge k^2 - k + 2$ for all $k - 1$ which is a prime power [Ir, Chu2, ChGra1], and $R_k(C_4) \ge k^2 + 2$ for odd prime power k [LaWo1]. The latter was extended to any prime power k in [Ling, LaMu]. For even number of colors k we have $R_k(C_4) \le k^2 + k$ [Boza8].

(c) Formulas for $R(C_m, C_n, C_k, C_l)$ for large m [EFRS1].

Bounds in (d)-(j) below cover different situations and each is interesting in some respect.

(d) $R_k(C_{2m}) \geq (k+1)m$ for odd k and $m \geq 2$, and
$R_k(C_{2m}) \geq (k+1)m-1$ for even k and $m \geq 2$ [DzNS].

(e) $R_k(C_{2m}) \geq 2(k-1)(m-1)+2$ [SunYXL].

(f) $R_k(C_{2m}) \geq k^2 + 2m - k$ for $2m \geq k+1$ and prime power k [SunYJLS].

(g) $R_k(C_{2m}) = \Theta(k^{m/(m-1)})$ for fixed $m = 2$, 3 and 5 [LiLih].

(h) $R_k(C_{2m}) \leq 201km$ for $k \leq 10^m/201m$ [ErdG].

(i) $R_k(C_{2m}) \leq 2km + o(m)$ for all fixed $k \geq 2$ [ŁucSS].

(j) $R_k(C_{2m}) \leq 2(k-c_k)m + o(m)$ for some small $c_k > 0$, for all fixed $k \geq 2$ [Sár2]. This was improved to an absolute constant $c = c_k = 1/4$ in [DavJR], and further to $c = 1/2$ in [KniSu]. See also 6.4.2.d.

(k) $R_k(C_5) < (18^k k!)^{1/2}/10$ [Li4].

(l) $2^k m < R_k(C_{2m+1}) \leq (k+2)!(2m+1)$ [BoEr].
Better upper bound $R_k(C_{2m+1}) < 2(k+2)!\,m$ was obtained in [ErdG].
Still better upper bound $R_k(C_{2m+1}) \leq (c^k k!)^{1/m}$, for some positive constant c,
if all Ramsey-critical colorings for C_{2m+1} are not far from regular, was obtained in [Li4].

(m) For each fixed $m \geq 3$, there exists a positive constant c such that for every $k \geq 3$, $R_k(C_{2m+1}) < c^{k-1}(k!)^{1/2+\delta}$, where δ is approaching 0 for large m [LinCh].

(n) $R_k(C_{2m+1}) \leq k2^k(2m+1) + o(m)$ for all fixed $k \geq 4$ [ŁucSS].

(o) Conjecture that $R_k(C_{2m+1}) = 2^k m + 1$ for all $m \geq 2$ was credited by several authors to Bondy and Erdős [BoEr], though only lower bound, not the conjecture, is in this paper. After more than 40 years, in 2016, Jenssen and Skokan [JenSk] posted a preprint on arXiv (which appeared in *Advances in Mathematics* in 2021) containing a proof of the conjecture for each fixed k with sufficiently large m. On the other hand, the work by Day and Johnson [DayJ] shows that the lower bound of the conjecture does not hold for each m and sufficiently large k.

(p) $R(C_n, C_{l_1}, \ldots, C_{l_k}) = 2^k(n-1)+1$ for all l_i's odd with $l_i > 2^i$, and sufficiently large n, and support for the conjecture that $R_k(C_n) = 2^{k-1}(n-1)+1$ for large odd n [AlBS].

(q) $R_l(C_{\leq l+1}) = 2l+3$ for all odd $l \geq 3$. For even l we have: $R_4(C_{\leq 5}) = 12$, $R_6(C_{\leq 7}) = 12$, and $R_l(C_{\leq l+1}) = 2l+3$ for $l = 8$, 10 and 12 [ZhuSWZ].

(r) Other asymptotic bounds for $R_k(C_n)$ [Bu1, GRS, ChGra2, Li4, LiLih, ŁucSS].

(s) Survey of multicolor cycle cases [Li3].

6.3.3. Cycles versus other graphs

(a) Some cases involving C_4:

$20 \le R(C_4, C_4, K_4) \le 21$	[DyDz1] [LidP]
$27 \le R(C_3, C_4, K_4) \le 29$	[DyDz1] [BoRa]
$52 \le R(C_4, K_4, K_4) \le 66$	[XSR1] [BoRa]
$34 \le R(C_4, C_4, C_4, K_4) \le 48$	[DyDz1] [LidP]
$43 \le R(C_3, C_4, C_4, K_4) \le 75$	[DyDz1] [BoRa]
$87 \le R(C_4, C_4, K_4, K_4) \le 177$	[XSR1] [BoRa]
$R(K_{1,3}, C_4, K_4) = 16$	[KlaM2]
$R(C_4, C_4, K_4 - e) = 16$	[DyDz1]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ExRe]

(b) Study of $R(C_n, K_{t_1}, \ldots, K_{t_k})$ and $R(C_n, K_{t_1, s_1}, \ldots, K_{t_k, s_k})$ for large n [EFRS1].

(c) $R(C_n, K_{t_1}, \ldots, K_{t_k}) = (n-1)(r-1)+1$ for $n \ge 4r+2$, where $r = R(K_{t_1}, \ldots, K_{t_k})$. This equality was obtained as a special case of more general results in [OmRa2]. Similar proof was presented later in [Mad]. Further, see items 6.6.f and 6.7.f.

(d) Study of asymptotics for $R(C_m, \ldots, C_m, K_n)$, in particular for any fixed number of colors $k \ge 4$ we have $R(C_4, C_4, \ldots, C_4, K_n) = \Theta(n^2/\log^2 n)$ [AlRö].

(e) Study of asymptotics for $R(C_{2m}, C_{2m}, K_n)$ for fixed m [AlRö, ShiuLL], in particular $R(C_4, C_4, K_n) = \Theta(n^2 \text{poly-}\log n)$ [AlRö].

(f) For fixed m and large n, $\Omega(n^{1+2/(2m-1)}/(\log n)^{4/(2m-1)}) \le R(C_{2m+1}, C_{2m+1}, K_n) \le O(n^{(1+1/m)^2}/(\log n)^{(2m+1)/m^2})$ [LiuLi5].

(g) Study of the general upper bound on $R(C_4, \ldots, C_4, K_{1,n})$, which for 3 colors implies $R(C_4, C_4, K_{1,n}) \le n+3+\lceil\sqrt{4n+5}\rceil$ [ZhaCC4]. This was extended to hold for wheel graphs as in $R(C_4, C_4, W_n) \le n+3+\lceil\sqrt{4n+5}\rceil$ for all $n \ge 56$ or n equal to 42, 48, 49, 50, 51 or 52. Furthermore, $R(C_4, C_4, W_{k^2-k}) \le k^2+k+2$ for $k \ge 9$, with equality for prime powers k [ZhaCC5].

(h) Lower and upper bounds on $R(C_4, \ldots, C_4, K_{1,n})$ [ZhaCC4, ZhaCC6, Boza8]. The equality $R(C_4, \ldots, C_4, K_{1,n}) = R(C_4, \ldots, C_4, W_{1,n})$ for large n was obtained in [ZhaCC6], note that it generalizes item 4.3.e.

(i) Upper bounds for $R(C_4, \ldots, C_4, G_1, \ldots, G_k)$, when certain constraints on graphs G_i are known, applicable in particular to stars [BoRa].

(j) $R(C_4, C_4, K_{1,n}) = 6, 8, 9, 11, 13, 14$ for $n = 1, 2, 3$ [ArKM], and $n = 4, 5, 6$ [ZhaCC4].

(k) Study of $R(H, H, C_n)$, for sufficiently large odd n, and for suitably defined balanced bipartite graphs H with small bandwidth [YouLin2].

(l) Study of $R(C_4, K_{1,m}, P_n)$ [ZhZC, SunSh].

(m) Monotone paths and cycles [Lef], see also 4.5.s.

(n) For combinations of C_3 and K_n see Sections 2.2, 3.2, 4.2, 6.1 and 6.2.

6.4. Paths, paths versus other graphs

In 2007, Gyárfás, Ruszinkó, Sárközy and Szemerédi [GyRSS] established that for all n large enough we have

$$R(P_n, P_n, P_n) = 2n - 2 + (n \bmod 2).$$

Faudree and Schelp [FS2] conjectured that the latter holds for all $n \geq 1$. It is true for $n \leq 9$ (see (c) below), and the first open case is that for P_{10}. The conjectured equality $R(C_{2m}, C_{2m}, C_{2m}) = 4m$ (see 6.3.1.a), whenever true, implies the above for three paths P_{2m+1} case [DyDR].

6.4.1. Three-color path and path-cycle cases

(a) $R(P_m, P_n, P_k) = m + \lfloor n/2 \rfloor + \lfloor k/2 \rfloor - 2$ for $m \geq 6(n+k)^2$ [FS2],
the equality holds asymptotically for $m \geq n \geq k$ with an extra term $o(m)$ [FiŁu1],
extensions of the range of m, n, k for which (a) holds were obtained in [Biel3].

(b) $R(P_3, P_m, P_n) = m + \lfloor n/2 \rfloor - 1$ for $m \geq n$ and $(m, n) \neq (3, 3), (4, 3)$ [MaORS2].

(c) $R_3(P_3) = 5$ [Ea1], $R_3(P_4) = 6$ [Ir],
$R(P_m, P_n, P_k) = 5$ for other $m{-}n{-}k$ combinations with $3 \leq m, n, k \leq 4$ [ArKM],
$R_3(P_5) = 9$ [YR1], $R_3(P_6) = 10$ [YR1], and $R_3(P_7) = 13$ [YY],
$R_3(P_8) = 14$, $R_3(P_9) = 17$ [DyDR].

(d) $R(P_4, P_4, P_{2n}) = 2n + 2$ for $n \geq 2$,
$R(P_5, P_5, P_5) = R(P_5, P_5, P_6) = 9$,
$R(P_5, P_5, P_n) = n + 2$ for $n \geq 7$,
$R(P_5, P_6, P_n) = R(P_4, P_6, P_n) = n + 3$ for $n \geq 6$,
$R(P_6, P_6, P_{2n}) = R(P_4, P_8, P_{2n}) = 2n + 4$ for $n \geq 14$ [OmRa1].

(e) $R(P_m, P_n, C_k) = 2n + 2\lfloor m/2 \rfloor - 3$ for large n and odd $m \geq 3$ [DzFi2],
improvements on the range of m, n, k [Biel3, Fid1].

(f) $R(P_3, P_3, C_m) = 5, 6, 6$, for $m = 3, 4$ [ArKM], 5,
$R(P_3, P_3, C_m) = m$ for $m \geq 6$ [Dzi2].
$R(P_3, P_4, C_m) = 7$ for $m = 3, 4$ [ArKM] and 5,
$R(P_3, P_4, C_m) = m + 1$ for $m \geq 6$ [Dzi2].
$R(P_4, P_4, C_m) = 9, 7, 9$ for $m = 3, 4$ [ArKM] and 5 [Dzi2],
$R(P_4, P_4, C_m) = m + 2$ for $m \geq 6$ [DzKP].

(g) $R(P_3, P_5, C_m) = 9, 7, 9, 7, 9$ for $m = 3, 4, 5, 6, 7$ [Dzi2, DzFi2],
$R(P_3, P_5, C_m) = m + 1$ for $m \geq 8$ [DzKP].
A table of $R(P_3, P_k, C_m)$ for all $3 \leq k \leq 8$ and $3 \leq m \leq 9$ [DzFi2].

(h) $R(P_4, P_5, C_m) = 11, 7, 11, 11$, and $m + 2$ for $m = 3, 4, 5, 7$ and $m \geq 23$,
$R(P_4, P_6, C_m) = 13, 8, 13, 13$, and $m + 3$ for $m = 3, 4, 5, 7$ and $m \geq 18$ [ShaXSP].

(i) $R(P_3, P_n, C_4) = n + 1$ for $n \geq 6$ [DzFi2],
$R(P_3, P_n, C_6) = n + 2$ for $n \geq 6$,
$R(P_3, P_n, C_8) = n + 3$ for $n \geq 7$ [Fid1],
$R(P_3, P_n, C_k) = 2n - 1$, and
$R(P_4, P_n, C_k) = 2n + 1$ for odd $k \geq 3$ and $n \geq k$ [DzFi2].

(j) $R(P_3, P_6, C_m) = m + 2$ for $m \geq 23$,
$R(P_6, P_6, C_m) = R(P_4, P_8, C_m) = m + 4$ for $m \geq 27$,
$R(P_6, P_7, C_m) = m + 4$ for $m \geq 57$,
$R(P_4, P_n, C_4) = R(P_5, P_n, C_4) = n + 2$ for $n \geq 5$ [OmRa1].

(k) $R(P_3, C_3, C_3) = 11$ [BuE3], $R(P_3, C_4, C_4) = 8$ [ArKM], $R(P_3, C_6, C_6) = 9$ [Dzi2],
$R(P_3, C_m, C_m) = R(C_m, C_m) = 2m - 1$ for odd $m \geq 5$ [DzKP] (for $m = 5, 7$ [Dzi2]),

(l) $R(P_3, C_n, C_m) = R(C_n, C_m)$ for $n \geq 7$ and odd m, $5 \leq m \leq n$, and
some values and bounds on $R(P_3, C_n, C_m)$ in other cases [Fid1].

(m) $R(P_3, C_3, C_4) = 8$ [ArKM], $R(P_3, C_3, C_5) = 9$, $R(P_3, C_3, C_6) = 11$,
$R(P_3, C_3, C_7) = 13$, $R(P_3, C_4, C_5) = 8$, $R(P_3, C_4, C_6) = 8$,
$R(P_3, C_4, C_7) = 8$, $R(P_3, C_5, C_6) = 11$, $R(P_3, C_5, C_7) = 13$ and
$R(P_3, C_6, C_7) = 11$ [Dzi2].

(n) $R(P_4, C_3, C_5) = 13$, $R(P_4, C_4, C_5) = 10$, $R(P_4, C_4, C_6) = 9$,
$R(P_4, C_5, C_5) = 13$, $R(P_4, C_6, C_6) = 10$ [SunSh].

(o) A formula for $R(P_m, P_n, C_k)$ for k large enough and m, n satisfying some constraints. In addition, some cases involving tK_2 instead of C_k are derived as side results [KhoDz].

(p) Study of $R(P_n, C_4, K_{1,m})$ [ZhZC, SunSh].

(q) Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].

(r) Lower and upper bounds on $R_3(P_m \cup K_{1,n})$ [ZhouLMW].

(s) $R(P_3, K_4 - e, K_4 - e) = 11$ [Ex7]. All colorings which can form any color neighborhood for the case $R_3(K_4 - e)$ (see Section 6.5) were found in [Piw2].

6.4.2. More colors

(a) $R_k(P_3) = k + 1 + (k \bmod 2)$, $R_k(2P_2) = k + 3$ for all $k \geq 1$ [Ir].

(b) $R_k(P_4) = 2k + c_k$ for all k and some $0 \leq c_k \leq 2$. If k is not divisible by 3 then $c_k = 3 - k \bmod 3$ [Ir]. Wallis [Wall] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t + 1$, for all $t \geq 2$. Independently, the case $R_k(P_4)$ for $k \neq 3^m$ was completed by Lindström in [Lind], and later Bierbrauer proved $R_{3^m}(P_4) = 2(3^m) + 1$ for all $m > 1$. $R_3(P_4) = 6$ [Ir].

(c) $R_3(P_5) = 9$ [YR1] and $R_4(P_5) = 11$ [LiuMS]. Furthermore, for all $k > 4$ we have $R_k(P_5) = 3k + c$, where $c = 0$ for k equal to 2 or 3 mod 4, $c = 1$ for $k = 0$ mod 4, and $c = 2$ for $k = 1$ mod 4 [LiuMS].

(d) $R_k(P_n) \le (k - c_k)n + o(n)$ for some small $c_k > 0$, for all fixed $k \ge 2$ [Sár2]. This was improved to an absolute constant $c = c_k = 1/4$ in [DavJR], and further to $c = 1/2$ in [KniSu]. See also 6.3.2.j/o.

(e) Formula for $R(P_{n_1}, \dots, P_{n_k})$ for large n_1 [FS2], and some extensions [Biel3].
Conjectures about $R(P_{n_1}, \dots, P_{n_k})$ when all or all but one of n_i's are even [OmRa1].

(f) Formulas for $R(P_{n_1}, \dots, P_{n_k}, C_m)$ for some cases, for large m [OmRa1].

(g) Formula for $R(n_1P_2, \dots, n_kP_2)$, in particular $R(nP_2, nP_2, nP_2) = 4n - 2$ [CocL1]. New proof with characterization of all critical graphs [XuYZ]. Note how close the latter is to $R(C_{2n}, C_{2n}, C_{2n}) = 4n$, and see an earlier item 6.3.1.b.

(h) Cockayne and Lorimer [CocL1] found the exact formula

$$R(n_1P_2, \dots, n_kP_2) = n_1 + 1 + \sum_{i=1}^{k}(n_i - 1), \text{ where } n_1 = \max\{n_1, \dots, n_k\}.$$

Later, Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \dots, n_kP_2)$. More general cases of the latter, with multiple copies of the complete graph, paths, stars and forests, were studied in [Stahl, LorSe, LorSo, GyRSS]. A special 3-color case $R(P_3, mP_2, nP_2) = 2m + n - 1$ for $m \ge n \ge 3$ is given in [MaORS2], and some other cases in [KhoDz]. The general case of multicolor combinations of stars and stripes is completed in [OmRR]. Ramsey numbers for path-matchings and covering designs, generalizing $R(n_1P_2, \dots, n_kP_2)$, are studied in [DeBGS].

(i) Multicolor cases for one large path or cycle involving small paths, cycles, complete and complete bipartite graphs [EFRS1].

(j) See Sections 6.5 and 8.2, especially [ArKM, BoDD], for a number of cases for triples of small graphs.

6.5. Special cases

(a) Denote $K_3 + e = K_4 - P_3$.

$R_3(K_3+e) = R_3(K_3)$ [$=17$]	[YR3, ArKM]
$R(K_3+e, K_3+e, K_4-e) = R(K_3, K_3, K_4-e) = 17$	[ShWR]
$R(K_3+e, K_3+e, K_5-P_3) = R(K_3, K_3, K_4)$ [$=30$]	[ShWR]
If $R_4(K_3) = 51$ then $R_4(K_3+e) = 52$, and if $R_4(K_3) > 51$ then $R_4(K_3+e) = R_4(K_3)$	[ShWR]

(b)

$R_3(K_4-e) = 28$	[Ex7] [LidP]
$R(P_3, K_4-e, K_4-e) = 11$	[Ex7], all colorings [Piw2]
$R(P_3, K_4-e, K_4) = 17$	[ArKM]
$R(P_3, K_4, K_4) = 35$	[BuE3], special case of 6.7.e

$472 \le R_3(K_6 - e)$	[HeLD]
$1102 \le R_3(K_7 - e)$	[HeLD], superseded by $1106 \le R_3(6)$ [Row3]

(c)

$21 \le R(K_3, K_4 - e, K_4 - e) \le 22$	[ShWR] [LidP]
$31 \le R(K_3, K_4, K_4 - e) \le 40$	[VO] [LidP]
$33 \le R(K_4, K_4 - e, K_4 - e) \le 47$	[ShWR] [LidP]
$55 \le R(K_4, K_4, K_4 - e) \le 94$	[Ea1] [LidP]

(d)

$R(C_4, P_4, K_4 - e) = 11$	[ArKM]
$R(C_4, P_4, K_4) = 14$	[BoDD]
$R(C_4, C_4, K_4 - e) = 16$	[DyDz1]
$R(C_4, K_3, K_4 - e) = 17$	[BoDD]
$R(C_4, K_4 - e, K_4 - e) = 19$	[BoDD]
$29 \le R(C_4, K_4, K_4 - e) \le 36$	[VO] [BoDD]
$52 \le R(C_4, K_4, K_4) \le 66$	[XSR1] [BoRa]

(e) For prime $p = 3q + 1$, if the cubic residues Paley graph Q_p contains no $K_k - e$, then $R_3(K_{k+1} - e) > 3p$ [HeLD]. The cases $k = 5$ and $k = 6$ give two bounds listed in (b). Also based off Paley graphs, several new lower bounds for $R_3(K_1 + G)$, and in particular for $R_3(B_n)$, were derived in [LinLS].

(f) If T_n is the set of all n-vertex trees (and all monochromatic n-vertex trees are avoided), then $R_3(T_n) = 2n - 2$ for even n, and $R_3(T_n) = 2n - 1$ for odd n [GeGy].

(g) $R_3(G_{a,b}) = (2 + o(1))ab$, where $G_{a,b}$ is the rectangular $a \times b$ grid graph. Lower and upper bounds on $R_3(G)$ for graphs G with small bandwidth and bounded $\Delta(G)$ [MoSST].

(h) See also Section 8.2 for pointers to cumulative data for three colors.

6.6. General results for special graphs

(a) Formulas for $R_k(G)$, where G is one of the graphs P_3, $2K_2$ and $K_{1,3}$, for all k, and for P_4 if k is not divisible by 3 [Ir]. For some details see Section 6.4.2.b.

(b) $tk^2 + 1 \le R_k(K_{2,t+1}) \le tk^2 + k + 2$, where the upper bound is general, and the lower bound holds when both t and k are prime powers [ChGra1, LaMu].

(c) $(m-1)\lfloor (k+1)/2 \rfloor < R_k(T_m) \le 2km + 1$ for any tree T_m with m edges [ErdG], see also [GRS]. The lower bound can be improved for special large k [ErdG, GRS]. The upper bound was improved to $R_k(T_m) < (m-1)(k + \sqrt{k(k-1)}) + 2$ in [GyTu].

(d) $k(\sqrt{m} - 1)/2 < R_k(F_m) < 4km$ for any forest F_m with m edges [ErdG], see [GRS]. See also pointers in other items below which involve special forests (like trees and stars).

(e) $R(S_1, \ldots, S_k) = n + \varepsilon$, where S_i's are arbitrary stars, $n = n(S_1) + \ldots + n(S_k) - 2k$, and we set $\varepsilon = 1$ if n is even and some $n(S_i)$ is odd, and $\varepsilon = 2$ otherwise [BuRo1]. See also [GauST, Par6]. Note that for graph G (here the set of edges in a given color), to avoid star $S = K_{1,n}$ is equivalent to have $\delta(G) < n$.

(f) Formula for $R(S_1, \ldots, S_k, K_n)$, where S_i's are arbitrary stars [Jaco]. It was generalized to a formula for $R(S_1, \ldots, S_k, K_{k_1}, \ldots, K_{k_r})$ expressed in terms of $R(k_1, \ldots, k_r)$ and star orders [BoCGR]. A much shorter proof of the latter was presented in [OmRa2]. Special cases for bistars [AlmHS], and bounds for stars and trees instead of stars [Bai].

(g) Formula for $R(S_1, \ldots, S_k, nK_2)$, where S_i's are arbitrary stars [CocL2], and a formula for $R(n_1K_2, \ldots, n_kK_2)$ [CocL1]. A new proof with characterization of all critical graphs [XuYZ]. See also cases involving P_2 in Section 6.4.2.

(h) Formula for $R(S_1, \ldots, S_k, G)$, where S_i's are stars and G is a tree [ZhZ1], or G is a cycle or wheel [RaeZ], for G of some orders depending on stars. Extension of these results to larger ranges of orders of G, and for G being a path [Wang1, ZhaHou]. Special cases when S_i's are trees and G is a wheel [RaeZ].

(i) Formulas for $R(S_1, \ldots, S_k)$, where each S_i's is a star or m_iK_2 [ZhZ2, ErdG, OmRR], formula for the case $R(S, mK_2, nK_2)$ [GySá2]. Several cases of formulas and asymptotics for $R_k(S(n,m))$, for sufficiently large n, where $S(m,n)$ is a double star [RuoS, Sár3]. The relation of $R_k(S(n,m))$ to the corresponding list Ramsey numbers [RuoS].

(j) Formula for $R(F, K_{k_1}, \ldots, K_{k_r})$ in terms of $R(K_{k_1}, \ldots, K_{k_r})$ and the size and structure of any forest F [KamRa]. This corrects a claim in an earlier version of [AlmBCL]. The latter studies the concept of p-goodness.

(k) Bounds on $R_k(G)$ for unicyclic graphs G of odd girth.
Some exact values for special graphs G, for $k = 3$ and $k = 4$ [KrRod].

(l) $R_k(K_{3,3}) = (1 + o(1))k^3$ [AlRóS].

(m) Bounds on $R_k(K_{s,t})$, in particular for $K_{2,2} = C_4$ and $K_{2,t}$ [ChGra1, AxFM]. Asymptotics of $R_k(K_{s,t})$ for fixed k and s [DoLi, LiTZ]. Upper bounds on $R_k(K_{s,t})$ [SunLi]. Upper bounds on multicolor numbers where all but the last color involve $K_{s,t}$ while the last color avoids a star or another bipartite graph [WaLL]. Upper bounds in the case when the last color is avoiding generalized books [LiLW]. Upper bounds in the case when the last color is avoiding fans [LiZZ].

(n) Asymptotics of $R(k, \ldots, k, n)$ in k, n and the number of colors [HeWi]. Exact asymptotics $R(K_{t,s}, K_{t,s}, K_{t,s}, K_m) = \Theta(m^t/\log^t m)$, for any fixed $t > 1$ and large $s \geq (t-1)! + 1$ [AlRö].

(o) Variety of asymptotic results on $R(K_{2,s}, \ldots, K_{2,s}, K_m)$ [LeMu]. Asymptotic upper bounds on $R(G, \ldots, G, H)$ in two cases: $G = K_{m,n}$, H is any nonempty graph, and $G = B_n$, $H = K_{m,n}$ [LiW].

(p) Bounds on $R_k(G)$ for trees, forests, stars and cycles [Bu1].
Bounds for trees $R_k(T)$ and forests $R_k(F)$ [ErdG, GRS, BierB, GyTu, Bra1, Bra2, SwPr].

(q) Study of the case $R(K_m, n_1P_2, \ldots, n_kP_2)$ [Lor]. Other similar results include $R(P_3, mK_2, nK_2) = 2m+n-1$ for $m \geq n \geq 3$ [MaORS2] and $R(S_n, nK_2, nK_2) = 3n-1$ [GySá2]. More general cases, with multiple copies of the complete graph, stars and forests, were investigated in [Stahl, LorSe, LorSo, GyRSS, OmRR]. See also Section 6.4.2.

(r) See Section 8.2, especially [ArKM, BoDD], for a number of cases for other small graphs, similar to those listed in Sections 6.3 and 6.4.

6.7. General results

(a) In 2001, Nešetřil and Rosenfeld [NeRo] in an overview paper withs numerous historical comments discuss the connections between the Ramsey numbers $R_r(3)$, Schur partitions and Shannon capacity of graphs.

(b) In 2020, the limit $\lim_{r\to\infty} R_r(3)^{1/r}$ was studied by Fox, Pach and Suk [FoxPS1] assuming a conjecture for multicolorings with bounded VC-dimension, and further for $\lim_{r\to\infty} R_r(k)^{1/r}$ when restricted to the so-called semi-algebraic colorings [FoxPS2].

(c) Szemerédi's Regularity Lemma [Szem] states that the vertices of every large graph can be partitioned into similar size parts so that the edges between these parts behave almost randomly. This lemma has been used extensively in various forms to prove the upper bounds, including those studied in [BenSk, GyRSS, GySS1, HaŁP1+, HaŁP2+, KoSS1, KoSS2].

(d) $R(m_1G_1, \ldots, m_kG_k) \leq R(G_1, \ldots, G_k) + \sum_{i=1}^{k} n(G_i)(m_i-1)$, exercise 8.3.28 in [West].

(e) If G is connected and $R(K_k, G) = (k-1)(n(G)-1)+1$, in particular if G is any n-vertex tree, then $R(K_{k_1}, \ldots, K_{k_r}, G) = (R(k_1, \ldots, k_r)-1)(n-1)+1$ [BuE3]. A generalization for connected $G_1, \ldots, G_n$ in place of G appeared in [Jaco].

(f) Conjecture that $R_3(H) \leq 2^{\Delta^{1+o(1)}}n$, where $\Delta = \Delta(H)$ [ConFS7].

(g) For connected graphs $G_1, \ldots, G_n$ with $s = R(G_1, \ldots, G_n)$ and $t = R(K_{k_1}, \ldots, K_{k_r})$, if $m \geq 2$ and $R(G_1, \ldots, G_n, K_m) = (s-1)(m-1)+1$, then $R(G_1, \ldots, G_n, K_{k_1}, \ldots, K_{k_r}) = (s-1)(t-1)+1$ [OmRa2]. The latter implies that $R(T_k, K_{k_1}, \ldots, K_{k_r}) = (k-1)(t-1)+1$ for any k-vertex tree T_k. The same result as in [OmRa2] was presented much later in [Mad]. It also generalizes a result in [BoCGR].

(h) If F, G, H are connected graphs then $R(F, G, H) \geq (R(F, G)-1)(\chi(H)-1) + \min\{R(F, G), s(H)\}$, where $s(G)$ is the chromatic surplus of G (see item [Bu2] in Section 5.16). This leads to several formulas and bounds for F and G being stars and/or trees when $H = K_n$ [ShiuLL].

(i) $R(K_{k_1}, \ldots, K_{k_r}, G_1, \ldots, G_s) \geq (R(k_1, \ldots, k_r)-1)(R(G_1, \ldots, G_s)-1)+1$ for arbitrary graphs $G_1, \ldots, G_s$ [Bev]. This generalizes 6.2.q, but is a special case of 6.7.g.

(j) Constructive bound $R(G_1, ..., G_{t^{n-1}}) \geq t^n + 1$ for decompositions of K_{t^n} [LaWo1, LaWo2].

(k) $R(G_1, \ldots, G_k) \leq 32\Delta k^{\Delta} n$, where $n \geq n(G_i)$ and $\Delta \geq \Delta(G_i)$ for all $1 \leq i \leq k$,
$R(G_1, \ldots, G_k) \leq k^{2k\Delta q} n$, where $q \geq \chi(G_i)$ for all $1 \leq i \leq k$ [FoxSu1].

(l) $R_k(G) \leq k^{6e(G)^{2/3}k}$ for all isolate-free graphs G and $k \geq 3$ [JoPe].
For the original two-color conjecture, now a theorem, see item 5.16.j [Erd4].

(m) $R_k(G) > (sk^{e(G)-1})^{1/n(G)}$, where s is the number of automorphisms of G [ChH3]. Other general bounds for $R_k(G)$ [ChH3, Par6].

(n) Study of $R(G_1, \ldots, G_k, G)$ for large sparse G [EFRS1, Bu3].

(o) Study of asymptotics for $R(H, \ldots, H, K_m)$, in particular when H is a fixed bipartite graph, and for $R(C_n, \ldots, C_n, K_m)$ [AlRö]. An improvement of the lower bound asymptotics for C_5 and C_7 was obtained in [XuGe]. See also Sections 6.3.3.d/e.

(p) Relations between the Shannon capacity of noisy communication channels and graph Ramsey numbers. A lower bound construction for $R_k(m)$ implying that supremum of the Shannon capacity over all graphs with bounded independence cannot be achieved by any finite graph power [XuR3]. For some other links between Shannon capacity and Ramsey numbers see Section 6 in [Ros2], and [Li2].

(q) See surveys listed in Section 8.

7. Hypergraph Numbers

7.1. Values and bounds for numbers

The only known value of a nontrivial classical Ramsey number for hypergraphs:

$$R(4,4;3) = 13 \quad \text{[MR1]}$$

There are exactly 434714 critical colorings on 12 points, none of which extend to a 2-coloring of all triples in $K_{13}-t$ without monochromatic K_4 [McK2]

The computer evaluation of $R(4,4;3)$ in 1991 consisted of an improvement of the upper bound from 15 to 13. This result followed an extensive theoretical study of this number by several authors [Gi4, Isb1, Sid1].

(a) $35 \le R(4, 5;3)$ [Dyb2]
$63 \le R(4, 6;3)$ [Dyb3]
$88 \le R(5, 5;3)$ [Dyb3]
$79 \le R(4,4,4;3)$ [Dyb3]
$35 \le R(5, 5;4)$ [Ex24]
$163 \le R(5,5,5;3)$ [BudHR1]

The last bound can be much improved to $7570 \le R(5,5,5;3)$ by using $88 \le R(5, 5;3)$ and a general constructive result in [BrBH], which yields $R_k(5;3) \ge 87^{2^{k-2}}$.

(b) $R(K_4-t, K_4-t;3) = 7$ [Ea2]
$R(K_4-t, K_4;3) = 8$ [Sob, Ex1, MR1]
$R(K_4-t, K_5-t;3) = 12$ [LidP]
$14 \le R(K_4-t, K_5;3) \le 16$ [Ex1] [LidP]
$13 \le R(K_4-t, K_4-t, K_4-t;3) \le 14$ [Ex1] [LidP]

(c) The first bound on $R(4, 5;3) \ge 24$ was obtained by Isbell [Isb2]. Shastri [Shas] gave a weak bound $R(5, 5;4) \ge 19$ (after a few improvements, now 35 in [Ex24]), nevertheless his lemmas, the stepping-up lemmas by Erdős and Hajnal (see [GRS, GrRö], also item 7.4.a), and others in [Ka3, Abb2, GRS, GrRö, HuSo, SonYL] can be used to derive better lower bounds for higher numbers.

(d) Several lower bound constructions for 3-uniform hypergraphs were presented in [HuSo]. Study of lower bounds on $R(p, q;4)$ can be found in [Song3] and [SonYL, Song4] (the latter two papers are almost the same in contents). Most of the concrete lower bounds in these papers can be easily improved by using the same techniques, but starting with better constructions for small parameters as listed above.

(e) $R(p, q;4) \ge 2R(p-1, q;4) - 1$ for $p, q > 4$, and
$R(p, q;4) \ge (p-1)R(p-1, q;4) - p + 2$ for $p \ge 5$, $q \ge 7$ [SonYL].
Lower bound asymptotics for $R(p, q;4)$ [SonLi].

(f) Recurrence relations in the form $R(p, q; r) \geq d(R(p-1, q; r)-1)+1$, where d depends on p, q and r, including the following: There exists $c \geq 25$, such that for k, $5 \leq k \leq c$, and any $p \geq k+2$ and $q \geq k+1$, we have $R(p, q; r) \geq (p-1)(R(p-1, q; r)-1)+1$ [Liu]. Such relations lead to the following bounds:

$R(5, 6; 4) \geq 67$, $R(6, 6; 4) \geq 133$, $R(7, 6; 4) \geq 661$,
$R(7, 7; 4) \geq 3961$, $R(8, 8; 4) \geq 194041$, $R(13, 6; 4) \geq 50689$,
$R(6, 6; 5) \geq 72$.

(g) $R(K_{1,1,c}, K_{1,1,c}; 3) = c+2$ for $2 \leq c \leq 4$, and a conjecture that this equality holds for all $c \geq 5$. The lower and upper bounds are closely related to the existence of appropriate BIBDs (balanced incopmplete block designs) with block size 3. The conference and journal versions of this work [MiPal] differ on some results. See also item 7.3.g.

(h) Lower bound asymptotics for $R(4, n; 3)$ [ConFS2],
lower bound asymptotics for $R(5, n; 4)$ [MuSuk2, MuSuk3], and
lower bound asymptotics for $R(6, n; 4)$ [MuSuk3].

(i) Lower and upper bounds on $R(K_4-t, K_n; 3)$ [ErdH, MuSuk2]. Extensions to *r-half-graph B^r*, where $B^3 = K_4-t$ [MuSuk2].

(j) Several constructive lower bounds for hypergraph numbers, including constructions which introduce a new color. In particular, they imply that $R_k(5;3)$ is equal to at least 82, 163, 131073, 262145 or 524289, for $k =$ 2, 3, 4, 5 and 6 colors, respectively [BudHR1]. Using 7.1.e and other known concrete lower bounds, $R(5,6;4) \geq 67$ and $R(4,4,5,5,5,5;3) \geq 17179869185$ are noted in [BudHP].

7.2. Cycles and paths

Definitions. $P_n^{r,s}$ is called an *s-path* in an r-uniform hypergraph H, if it consists of n hyperedges $\{e_1, ..., e_n\}$ in $E(H)$, such that $|e_i \cap e_{i+1}| = s$ for all $1 \leq i < n$, and all other vertices in e_j's are distinct [Peng]. An *s-cycle* $C_n^{r,s}$ is defined analogously. Several authors use the terms of *loose* paths and *loose* cycles, which are 1-path and 1-cycles, and *tight* paths and *tight* cycles, the latter most often for 3-uniform hypergraphs when they are 2-paths and 2-cycles, respectively. A 3-uniform *Berge* cycle is formed by n distinct vertices, such that all consecutive pairs of vertices are in an edge of the cycle, and all of the cycle edges are distinct. Berge cycles are not determined uniquely.

In the following items (b) to (i), when $r = 3$ or r is implied by the context, we write C_n and P_n for the r-uniform loose cycles and paths, $C_n^{r,1}$ and $P_n^{r,1}$, respectively. In other cases special comments are added.

Two colors

(a) Tetrahedron is formed by four triples on the set of four points. The Ramsey number of tetrahedron is $R(4, 4; 3) = 13$ [MR1].

(b) For loose cycles and paths, $R(C_3, C_3; 3) = 7$, $R(C_4, C_4; 3) = 9$, and for the r-uniform case we have in general $R(P_3, P_3; r) = R(P_3, C_3; r) = R(C_3, C_3; r) + 1 = 3r - 1$ and $R(P_4, P_4; r) = R(P_4, C_4; r) = R(C_4, C_4; r) + 1 = 4r - 2$, for $r \geq 3$. These results and discussion of several related cases were presented in [GyRa].

(c) $R(P_m, P_n; 3) = R(C_m, C_n; 3) + 1 = R(P_m, C_n; 3) = 2m + \lfloor (n+1)/2 \rfloor$, for all $m \geq n$, and $R(C_m, P_n; 3) = 2m + \lfloor (n-1)/2 \rfloor$, for $m > n$ [MaORS1, OmSh1]. The formulas for 3-uniform two-color cases of some loose paths versus stars are derived in [ZhCh].

(d) For loose cycles, $R(C_{2n}, C_{2n}; 3) > 5n - 2$ and $R(C_{2n+1}, C_{2n+1}; 3) > 5n + 1$, and asymptotically these lower bounds are tight [HaŁP1+]. Generalizations to r-uniform hypergraphs and graphs other than cycles appeared in [GySS1].

(e) For loose cycles, $R(C_n, C_n; r) = (r-1)n + \lfloor (n-1)/2 \rfloor$ for $n \geq 2$, $r \geq 8$ [OmSh2], and it also holds for $r = 4$ [OmSh3]. Further extensions to off-diagonal cases as in (c) are obtained in [OmSh4]. Based on these results, it was conjectured that for $n \geq m \geq 3$ and $r \geq 3$, we have $R(C_n, C_m; r) = (r-1)n + \lfloor (m-1)/2 \rfloor$. In [Shah], the known cases of this conjecture are discussed, and it is shown that it holds for $r = 5$ with large n.

(f) For tight cycles, $R(C_{3n}, C_{3n}; 3) \approx 4n$ and $R(C_{3n+i}, C_{3n+i}; 3) \approx 6n$ for $i = 1$ or 2, and for tight paths $R(P_n, P_n; 3) \approx 4n/3$ [HaŁP2+]. Some related results are discussed in [PoRRS]. Also for tight paths and cycles, but 4-uniform, we have $R(P_n, P_n; 4) \approx 5n/4$ and $R(C_{4n}, C_{4n}; 4) \approx 5n$ [LoPfe].

(g) Exact values for Ramsey numbers involving s-paths for even r and $s = r/2$, in particular for $P_n^{r,s}$ versus $P_3^{r,s}$ and $P_4^{r,s}$, when this value is $(n+1)s + 1$ [Peng].

(h) For 3-uniform Berge cycles and two colors, we have $R(C_n, C_n; 3) = n$ for $n \geq 5$ [GyLSS]. Some results for Berge-cycle versus Berge-complete numbers are obtained in [MahS], and the asymptotics of Berge-cycles versus complete hypergrap is derived in [NieVer].

(i) Lower and upper asymptotic bounds for $R(C_3^{3,1}, K_m; 3)$ and $R(C_3^{r,1}, K_m; r)$ [KosMV2].

(j) Lower and upper asymptotic bounds for $R(C_s, K_m; 3)$ for tight cycles C_s [MuR].
An improvement of the upper bound from the latter [Mub1].

(k) Gyárfás, Sárközy and Szemerédi proved that, for sufficiently large n, every 2-coloring of the edges of the complete 4-uniform hypergraph K_n contains a monochromatic 3-tight Berge cycle C_n [GySS2]. Exact formulas and bounds for Berge-K_n hypergraphs, including higher uniformity r [SaTWZ].

(l) Upper bounds on asymptotics of $R(C_n^{r,1}, K_m; r)$ for even and odd n [ColGJ]. Improvements of the results from the latter, in particular for the case of $n = 5$ and $r = 3$, and for general n [Mér].

(m) Summary of known values and ranges for hypergraph numbers for loose paths (and some other trees) versus complete hypergraphs, $R(P_m, K_n; 3)$, for $n \leq 10$ and odd m [BudP].

(n) Study of the growth rate of $R(P_m, K_n; r)$ for tight paths P_m with $m \geq r+3$, and links between the growth of $R(P_{r+1}, K_n; r)$ and $R(n, n; r)$ [MuSuk1]. The correct tower growth rate for ordered tight paths versus cliques [Mub2].

(o) Study of $R(G, nH; r)$ and $R(mG, nH; r)$ for G and H being loose/tight path, cycles and stars, including several exact results for large m or n [OmRa3]. The case of loose t-tight paths versus stars and some tripartite hypergraphs is explored in [BudHR2].

(p) Let F be the Fano plane, seen as a 3-uniform hypergraph of 7 hyperedges. If P_n and C_n are tight path and cycle on n vertices, respectively, then for sufficiently large n we have $R(P_n, F; 3) = 2n-1$ and $R(C_n, F; 3) = 2n-1$ [BalCSW].

More colors

(q) For loose cycles, $R_3(C_3; 3) = R(C_3, C_3, C_3; 3) = 8$, and in general for $k \geq 4$ colors Gyárfás and Raeisi established the bounds $k+5 \leq R_k(C_3; 3) \leq 3k$ [GyRa].

(r) For loose paths, we have $R_3(P_3; 3) = 9$ and $10 \leq R_4(P_3; 3) \leq 12$ [Jack]. This was improved to $R_k(P_3; 3) = k+6$ for all $2 \leq k \leq 9$ [JacPR, PoRu], and extended to $k = 10$ [Pol]. The general upper bound $R_k(P_3; 3) \leq 2k + \sqrt{18k+1} + 2$ was obtained in [ŁuPo1], then improved to $R_k(P_3; 3) \leq 1.975k + 7\sqrt{k} + 2$ [ŁuPo2], and then further improved to $R_k(P_3; 3) \leq 1.546k$ for large k [BohZ]. For the *messy path* $M_3 = \{abc, bcd, def\}$, we have $R_k(M_3; 3) \leq 1.6k$ for large k [BohZ].

(s) The case of $R_k(P_3; r)$ for loose path was asymptotically solved in [ŁuPR]. A general upper bound $R_k(P_n; r) \leq (r-1)kn$, for all $k, r \geq 2$ and $n \geq 3$, was obtained in [DuRu]. Do not be confused by notation in some of the papers when their bounds are expressed in terms of r colors and k-uniform paths (in contrast to k-colors and r-uniform used in this survey).

(t) For tight paths P_{m_i}, study of the growth rate of $R(P_{m_1}, ..., P_{m_k}, K_m; r)$ [MuSuk1].

(u) For 3-uniform Berge cycles, we have $R_3(C_n; 3) = (1+o(1))5n/4$ [GySá1], and also $R(C_n, C_n, C_3; 3) = n+1$ [MahS]. Some special cases for r-uniform hypergraphs with respect to Berge cycles were studied in [GyLSS], and for small uniformity r in [DeMST]. See also 7.3.l and 7.4.f.

(v) Study of Turán and Ramsey numbers of sets of minimal 3-uniform paths of length 4 for up to 4 colors [HanPR]. Minimality of path here means that there are no redundant edge intersections, in particular no vertex belongs to more than two edges.

7.3. General results for 3-uniform hypergraphs

(a) $2^{cn^2} < R(n, n; 3) < 2^{2^n}$ is credited to Erdős, Hajnal and Rado (see [ChGra2] p. 30).

(b) For some a, b the numbers $R(m, a, b; 3)$ are at least exponential in m [AbbS].

(c) Improved lower and upper asymptotics for $R(s, n; 3)$ for fixed s and large n, proof of related Erdős and Hajnal conjecture on the growth of $R(4, n; 3)$, and the lower bound $2^{n^{c \ln n}} < R(n, n, n; 3)$ [ConFS2].

(d) Let S_n denote the 3-uniform star on $n+1$ vertices consisting of all $n(n-1)/2$ triangles sharing one vertex. Then we have $2^{c\log^2 n} \le R(K_4, S_n; 3) \le 2^{c' n^{2/3} \log n}$, for some positive constants c, c' [ConFH+]. Study of the rate of growth of $R(K_n, H; 3)$ for fixed hypergraph H [ConFG+].

(e) The hedgehog H_t is a 3-uniform hypergraph with $t + t(t-1)/2$ vertices such that for every (i, j) with $1 \le i < j \le t$ there exists a unique vertex $k > t$ such that ijk is an edge, and H_t has no other edges. Conlon, Fox and Rödl studied the bounds on $R_k(H_t; 3)$ for $2 \le k \le 4$ and large t [ConFR]. The hypergraphs H_t constitute the first family of hypergraphs whose Ramsey numbers show a strong dependence on the number of colors: their 2-color Ramsey numbers grow polynomially in t, while in the 4-color case they grow exponentially. $R_k(H_t; 3) = O(t^2 \ln t)$ was obtained in [FoxLi].

(f) $R(G, G; 3) \le cn(H)$ for some constant c depending only on the maximum degree of a 3-uniform hypergraph H [CooFKO1, NaORS]. Similar results were proved for r-uniform hypergraphs in [KüCFO, Ishi, CooFKO2, ConFS1], see also item 7.4.i.

(g) Asymptotic lower bounds for $R(K_{a,b,c}, K_{a,b,c}; 3)$, where $K_{a,b,c}$ is formed by all abc triples on sets of orders a, b, c. The conference and journal versions of this work [MiPal] differ on some results. See also item 7.1.g.

(h) Asymptotic or exact values of $R_k(H; 3)$ when H is a bow $\{abc, ade\}$, kite $\{abc, abd\}$, tight path $P_3^{3,2} = \{abc, bcd, cde\}$, or windmill $\{abc, bde, cef, bce\}$, and a special concrete case for 6-color kite, $R_6(kite; 3) = 8$.

(i) $R_k(K_3; 2) \le R_{4k}(K_4 - t; 3) \le R_{4k}(K_3; 2) + 1$ [AxGLM].

(j) Variety of general lower bound constructions for 3-uniform complete or complete missing one hyperedge hypergraphs from liftings of graphs, for two and more colors. For example, we have $R(K_{2s_1-1} - t, K_{2s_2-1}, K_{2s_3-1}; 3) \ge R(s_1, s_2, s_3)$ [BudHMP] and $R(K_5, K_{43} - t, K_{43} - t, K_{43} - t; 3) > 1257480$ [BudHLS].

(k) If G is a 3-uniform H-free hypergraph, then G contains a complete or empty tripartite subgraph with parts of order $(\log n(H))^{c+1/2}$, where $c > 0$ depends only on H. Furthermore, for $k \ge 4$ no analogue of it can hold for k-uniform hypergraphs [ConFS5].

(l) Study of 3-uniform Berge-G graphs in r colors: asymptotic lower and upper bounds, and several exact values for small r with $G = K_3$ or $G = K_4$. Some asymptotics in the nonuniform case [AxGy]. This extends the results in 7.2.h [GyLSS] and 7.2.u [GySá1].

(m) Upper bounds on $R_k(H; 3)$ for complete multipartite 3-uniform hypergraphs H, a 4-color case, and some other general and special cases [ConFS1, ConFS2, ConFS3]. $R_k(H; 3)$ ranges from $\sqrt{6k}(1 + o(1))$ to double exponential in k [AxGLM]. Determination when $R_k(H; 3)$ behaves polynomially, exponentially or double-exponentially in k [BraFS].

7.4. General results

(a) If $R(n, n; r) > m$, then we have $R(2n + r - 4, 2n + r - 4; r + 1) > 2^m$ for all $n > r \geq 3$ (see [GRS] p. 106). This is the so-called stepping-up lemma, usually credited to Erdős and Hajnal. An improvement of the stepping-up lemma implying better lower bounds for a few types of hypergraph Ramsey numbers were obtained by Conlon, Fox and Sudakov [ConFS6].

(b) For k colors, lower bounds on $R_k(n; r)$ are discussed in [AbbW, DuLR, ConFS6, AxGLM, JaKSY]. In particular, if $R_k(n; r) > m$, then we have $R_{k+3}(n + 1; r + 1) > 2^m$ for $r \geq 3$ [JaKSY].

(c) General lower bounds for large number of colors were given in an early paper by Hirschfeld [Hir], and some of them were later improved in [AbbL].

(d) Lower and upper asymptotics of $R(s, n; k)$ for fixed s [ConFS2, MuSuk2, MuSuk3].

(e) Exact and asymptotic results generalizing 7.2.d/e to r-uniform case for cycles, and 2- and 3-color cases for all r-uniform diamond matchings [GySS1]. Connections between the existence of large sets of combinatorial $t-$designs to some bounds on Ramsey-type problems [Gy].

(f) Exact formulas and bounds for Berge-K_n hypergraphs, including multiple colors [AxGy] and higher uniformity r [AxGy, SaTWZ]. Progress on the conjecture that every $(r-1)$-coloring of K_n^r, for fixed r and large n, contains a monochromatic Hamiltonian Berge cycle [MaOm2]. Determination of some cases of uniformity r, number of colors and G, for which the Ramsey number of Berge-G is superlinear [Gerbn]. Further study of multicolor Ramsey numbers for such Berge-G hypergraphs, with some equalities and improved asymptotics, were obtained in [GerMOV, Pálv].

(g) Exact order of growth of the multicolor numbers for Berge cycles of length 4, 5, 6, 7, 10 and 11 for small uniformities, and for multicolor cases of Berge-$K_{a,b}$ for certain a, b and r, were derived in [DeMST].

(h) Study of $R(G, nH; r)$ and $R(mG, nH; r)$ for loose/tight path and cycles (possibly with some additions), stars, r-partite hypergraphs, including several exact results for large m or n [OmRa3].

(i) $R(H, H; r) \leq cn(H)^{1+\varepsilon}$, for some constant $c = c(\Delta, r, \varepsilon)$ depending only on the maximum degree of H, r and $\varepsilon > 0$ [KoRö3]. The proofs of the linear bound $cn(H)$ were obtained independently in [KüCFO] and [Ishi], the latter including the multicolor case, and then without regularity lemma in [ConFS1]. More discussion of lower and upper bounds for various cases can be found in [ConFS1, ConFS2, ConFS3, CooFKO2].

(j) Let T_r be an r-uniform hypergraph with r edges containing a fixed $(r-1)$-vertex set S and the $(r+1)$-st edge intersecting all former edges in one vertex outside S. Then $R(T_r, K_t; r) = O(t^r / \log t)$ [KosMV1].

(k) Study of tree-star and tree-complete cases of Ramsey numbers for r-uniform hypergraphs. Several bounds and equalities for special cases [BudHR1, BudCli]. This was posed and explored as a problem of which trees are Ramsey n-good hypergraphs [BudP]. Further results towards the conjecture that all r-uniform trees are n-good [BudCli].

Study of the Ramsey numbers of disjoint union of H-good hypergraphs [RaeK].

(l) Let $H^r(s, t)$ be the complete r-partite r-uniform hypergraph with $r-2$ parts of size 1, one part of size s, and one part of size t (for example, for $r=2$ it is the same as $K_{s,t}$). For the multicolor numbers, Lazebnik and Mubayi [LaMu] proved that

$$tk^2 - k + 1 \leq R_k(H^r(2, t+1); r) \leq tk^2 + k + r,$$

where the lower bound holds when both t and k are prime powers. For the general case of $H^r(s, t)$, more bounds are presented in [LaMu].

(m) $R_k(H; r)$ is polynomial in k when a fixed r-uniform H is r-partite, otherwise it is at least exponential in k [AxGLM].

(n) Grolmusz [Grol1] generalized the classical constructive lower bound by Frankl and Wilson [FraWi] (item 2.4.7) to more colors and to hypergraphs [Grol3].

(o) Lower and upper asymptotics, and other theoretical results on hypergraph numbers, are discussed in [GrRö, GRS, ConFS1, ConFS2, ConFS3, ConFS7, Song8, MuSuk1, MuSuk2, MuSuk3]. An extensive overview of progress and open problems in hypergraph Ramsey theory by Mubayi and Suk was compiled in 2018 [MuSuk4].

8. Cumulative Data, Books and Surveys

8.1. Cumulative data for two colors

(a) $R(G, G)$ for all graphs G without isolates on at most 4 vertices [ChH1].

(b) $R(G, H)$ for all graphs G and H without isolates on at most 4 vertices [ChH2].

(c) $R(G, H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries [Clan], now all solved, see Section 5.11. All critical colorings for the isolate-free graphs G and H studied in [Clan] were found in [He4].

(d) $R(G, G)$ for all graphs G without isolates and with at most 6 edges [Bu4].

(e) $R(G, G)$ for all graphs G without isolates and with at most 7 edges [He1].

(f) $R(G, G)$ for all graphs G on 5 vertices and with 7 or 8 edges [HaMe2].

(g) $R(G, H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries [He2]. Only 2 cases are still open, see 5.11 and the paragraph at the end of this section.

(h) Tables of $R(G, H)$ for most connected graphs on up to 5 vertices and $R(G, G)$ for all isolate-free graphs with up to 7 edges [ReWi].

(i) $R(G, H)$ for all disconnected isolate-free graphs H on at most 6 vertices versus all G on at most 5 vertices, except 3 cases [LoM5]. Missing cases were completed in [KroMe].

(j) $R(G, H)$ for some G on 5 vertices versus all connected graphs on 6 vertices [LoM6].

(k) $R(G, H)$ for $G = K_{1,3} + e$ and $G = K_4 - e$ versus all connected graphs H on 6 vertices, except $R(K_4 - e, K_6)$ [HoMe]. The result $R(K_4 - e, K_6) = 21$ was claimed by McNamara [McN, unpublished], now confirmed in [ShWR].

(l) $R(G, H)$ for some graphs G with 4 vertices versus all graphs H with 7 vertices [Boza4].

(m) $R(G, T)$ for all connected graphs G with $n(G) \leq 5$, and almost all trees T [FRS4].

(n) $R(T_m, T_n)$ for $6 \leq m, n \leq 8$, for k-vertex trees T_k [RanMCG].

(o) $R(K_3, G)$ for all connected graphs G on 6 vertices [FRS1].

(p) $R(K_3, G)$ for all connected graphs G on 7 vertices [Jin].
Some errors in the latter were found [SchSch1].

(q) $R(S, G)$ for stars S versus almost all connected graphs G on 6 vertices [LoM7]. This was extended to $R(T, G)$ for most non-star trees T, in particular for all trees on at most 5 vertices versus all connected graphs G on at most 6 vertices [LoM10]. The values and bounds for still missing cases are presented in [LoM8, LoM9].

(r) Formulas for $R(nK_3, mG)$ for all G of order 4 without isolates [Zeng].

(s) $R(K_3, G)$ for all connected graphs G on at most 8 vertices [Brin]. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KlaM1].

(t) $R(K_3, G)$ for all connected graphs G on 9 vertices [BrBH1, BrBH2].

(u) $R(K_3, G)$ for all graphs G on 10 vertices, except 10 cases [BrGS].
Three of the open cases, including $G = K_{10} - e$, were solved [GoR2].

(v) $R(C_4, G)$ for all graphs G on at most 6 vertices [JR3].
This work was followed by two errata listed in the references.

(w) $R(C_5, G)$ for all graphs G on at most 6 vertices [JR4].

(x) $R(C_6, G)$ for all graphs G on at most 5 vertices [JR2].

(y) $R(K_{2,n}, K_{2,m})$ for all $2 \le n,\ m \le 10$ except 8 cases, for which lower and upper bounds are given [LoM3]. Further data for other complete bipartite graphs are gathered in Section 3.3 and [LoMe4].

(z) All best lower bounds up to 102 from cyclic graphs. Formulas for best cyclic lower bounds for paths and cycles, and values for small complete graphs and for graphs with up to five vertices [HaKr1].

Chvátal and Harary [ChH1, ChH2] formulated several simple but very useful observations on how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved (Section 5.11). Out of 7 open cases in [He2] 5 have been solved, including $R(4, 5) = R(G_{19}, G_{23}) = 25$ and other cases listed in Section 5.11. The 2 cases still open are for K_5 versus K_5 (Section 2.1) and K_5 versus $K_5 - e$ (Section 3.1). Many extremal and other Ramsey graphs for various parameters are available at [BrCGM, McK1, Ex18, Fuj1], see Section 8.3 below.

8.2. Cumulative data for three colors

(a) $R_3(G)$ for all graphs G with at most 4 edges and no isolates [YR3].

(b) $R_3(G)$ for all graphs G with 5 edges and no isolates, except $K_4 - e$ [YR1].
This last open case is now solved, namely, we have $R_3(K_4 - e) = 28$ [Ex7][LidP].

(c) $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases [YY].

(d) $R(F, G, H)$ for many triples of isolate-free graphs with at most 4 vertices [ArKM].
Some of the missing cases completed in [KlaM2].

(e) Extension of [ArKM] to most triples of graphs with at most 4 vertices [BoDD].

(f) $R(P_3, P_k, C_m)$ for all $3 \le k \le 8$ and $3 \le m \le 9$ [DzFi2].

8.3. Electronic Resources

(a) W. Gasarch [Gas] maintains a website gathering over 60 pointers to literature on applications of Ramsey theory in computer science, and in particular logic, complexity theory and algorithms, http://www.cs.umd.edu/~gasarch/TOPICS/ramsey/ramsey.html.

(b) Many of the Ramsey graph constructions found by G. Exoo [Ex1-Ex25] are posted at http://cs.indstate.edu/ge/RAMSEY.

(c) G. Brinkmann, K. Coolsaet, J. Goedgebeur and H. Mélot, *House of Graphs: A database of interesting graphs* [BrCGM], http://hog.grinvin.org.

(d) B.D. McKay, presents some graphs related to classical Ramsey numbers [McK1], http://cs.anu.edu.au/people/bdm/data/ramsey.html.

(e) Set of Ramsey problems with comments and references by a team of students of Fan Chung, University of California San Diego [UCSD], *Erdős' Problems on Graphs, Ramsey Theory,* http://www.math.ucsd.edu/~erdosproblems/RamseyTheory.html (2010-2012).

(f) H. Fujita, collection of Ramsey graph constructions [Fuj1], http://opal.inf.kyushu-u.ac.jp/~fujita/ramsey.html, (2014-2017).

(g) M. Rubey, an electronic GUI resource for values of some small Ramsey numbers [Rub], http://www.findstat.org/StatisticsDatabase/St000479.

(h) S. Van Overberghe, Ramsey graph constructions associated with MS Thesis, Ghent University, Belgium, 2020 [VO], https://github.com/Steven-VO/circulant-Ramsey.

(i) A.E. Brouwer, Parameters of Strongly Regular Graphs [Brou], used mainly in 3.1.d and 5.3.h, https://www.win.tue.nl/~aeb/graphs/srg/srgtab.html.

8.4. Books and Surveys

Books and special works

(1980) *Ramsey Theory* by R.L. Graham, B.L. Rothschild and J.H. Spencer [GRS], first edition 1980, second edition 1990, paperback of the second edition 2013.

(1983) Special volume of the *Journal of Graph Theory* [JGT].

(1996) A chapter in *Handbook of Combinatorics* by J. Nešetřil [Neš].

(1997) Among 114 open problems and conjectures of Paul Erdős, presented and commented by F.R.K. Chung, 31 are concerned directly with Ramsey numbers [Chu4]. 216 references are given. An extended version of this work was prepared jointly with R.L. Graham [ChGra2] in 1998.

(2001) An extensive chapter on Ramsey theory in a widely used student textbook and researcher's guide of graph theory by D. West [West].

(2002) *Ramsey Theory and Paul Erdős* by R.L. Graham and J. Nešetřil [GrNe].

(2003) Special issue of *Combinatorics, Probability and Computing* [CoPC].

(2004) *Ramsey Theory on the Integers* by B. Landman and A. Robertson [LaRo], first edition 2004, second edition 2014.

(2009) History, results and people of Ramsey theory. *The mathematical coloring book, mathematics of coloring and the colorful life of its creators* by A. Soifer [Soi1].

(2011) *Ramsey Theory. Yesterday, Today and Tomorrow,* a special volume in the series *Progress in Mathematics* [Soi2]. A survey of Ramsey numbers involving cycles by this author is included there [Ra4].

(2015) *Recent Developments in Graph Ramsey Theory* by D. Conlon, J. Fox and B. Sudakov [ConFS7].

(2015) *Rudiments of Ramsey Theory,* a new edition of the classics by R.L. Graham and S. Butler [GrBu].

(2018) *Ramsey Theory, Unsolved Problems and Results* by Xiaodong Xu, Meilian Liang and Haipeng Luo [XuLL].

(2021) *The Discrete Mathematical Charms of Paul Erdős: A Simple Introduction,* by Vašek Chvátal [Chv2].

(2022) *Elementary Methods of Graph Ramsey Theory,* an extensive presentation of the area by Yusheng Li and Qizhong Lin [LiLin].

(2024) *The New Mathematical Coloring Book, Mathematics of Coloring and the Colorful Life of Its Creators,* greatly extended and revised first edition of the 2009 coloring book [Soi1] by Alexander Soifer [Soi3].

Surveys and Overviews

(1974) A general survey of results in Ramsey graph theory by S.A. Burr [Bu1]

(1978) A general survey of results in Ramsey graph theory by T.D. Parsons [Par6]

(1980) Survey of results and new problems on multiplicities and Ramsey multiplicities by S.A. Burr and V. Rosta [BuRo3]

(1981) Summary of progress by Frank Harary [Har2]

(1983) A survey of bounds and values by F.R.K. Chung and C.M. Grinstead [ChGri]

(1984) A review of Ramsey graph theory for newcomers by F.S. Roberts [Rob1]

(1987) An overview of progress so far and plans for the future, *What Can We Hope to Accomplish in Generalized Ramsey Theory?* by S. Burr [Bu7]

(1987) Survey of asymptotic problems by R.L. Graham and V. Rödl [GrRö]

(1991) Survey by R.J. Faudree, C.C. Rousseau and R.H. Schelp of graph goodness results, i.e. conditions for the formula $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ [FRS5]

(1996) Survey of zero-sum Ramsey theory by Y. Caro [Caro]

(2004) Dynamic survey of Ramsey theory applications by V. Rosta [Ros2]. A website maintained by W. Gasarch [Gas] gathers over 60 pointers to literature on applications of Ramsey theory in computer science.

(2010) *Hypergraph Ramsey Numbers* by D. Conlon, J. Fox and B. Sudakov [ConFS2].

(2013) *Problems in Graph Theory from Memphis,* "a summary of problems and results coming out of the 20 year collaboration between Paul Erdős and the authors", by R.J. Faudree, C.C. Rousseau and R.H. Schelp [FRS6].

(2016) *On Some Open Questions for Ramsey and Folkman Numbers* [XuR4].

(2018) *A Survey of Hypergraph Ramsey Problems,* by D. Mubayi and A. Suk [MuSuk4].

(2020) *An Introduction to Ramsey's Theorem* by A. Tripathi [Tri].

(2021) *New Directions in Ramsey Theory* by G. Chartrand and P. Zhang [ChaZ].

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par6] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Har2], where he also gathered many references including seven to other early surveys of this area. More than three decades ago, Chung and Grinstead in their survey paper [ChGri] gave much less data than in this work, but they included a broad discussion of different methods used in Ramsey computations in the classical case. S. A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu7] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS5, Neš, Chu4, ChGra2, ConFS7], though these focus on asymptotic theory not on the numbers themselves. A very welcome addition is the 2004 compilation of applications of Ramsey theory by V. Rosta [Ros2]. This survey could not be complete without recommending special volumes of the *Journal of Graph Theory* [JGT, 1983] and *Combinatorics, Probability and Computing* [CoPC, 2003], which, besides a number of research papers, include historical notes and present to us Frank P. Ramsey (1903-1930) as a person. Read a colorful book by A. Soifer [Soi1, 2009] on history and results in Ramsey theory, followed by a collection of essays and technical papers based on presentations from the 2009 Ramsey theory workshop at DIMACS [Soi2, 2011]. A 70-page long paper from 2015, entitled *Recent Developments in Graph Ramsey Theory,* by D. Conlon, J. Fox and B. Sudakov [ConFS7] documents in details what the title says. Finally, since the previous revision of this survey, some new and revised books appeared: in 2021 by V. Chvátal [Chv2], in 2022 by Li and Lin [LiLin], and in 2024 an extended and revised book by Soifer [Soi3].

The historical perspective and, in particular, the timeline of progress on prior best bounds, can be obtained by comparing all the previous versions of this survey since 1994 at the author's website http://www.cs.rit.edu/~spr/ElJC/eline.html or at the portal of the *ElJC* journal http://www.combinatorics.org/ojs/index.php/eljc/issue/view/Surveys.

9. Concluding Remarks

9.1. Exclusions

This compilation does not include much information on numerous variations of Ramsey numbers, nor related topics, like

anti-Ramsey numbers,
ascending Ramsey index,
avoiding sets of graphs in some colors,
barycentric Ramsey numbers,
bipartite Ramsey numbers,
blowup Ramsey numbers,
chromatic Ramsey numbers,
class Ramsey numbers,
complementary Ramsey numbers,
connected Ramsey numbers,
cover Ramsey numbers,
defective Ramsey numbers,
degree Ramsey numbers,
directed Ramsey numbers,
distance Ramsey numbers,
edge-chromatic Ramsey numbers,
edge-ordered Ramsey numbers,
Folkman numbers,
Gallai-Ramsey numbers,
generalized Ramsey numbers,
induced Ramsey numbers,
irredundant Ramsey numbers,
list Ramsey numbers,
local Ramsey numbers,
k-Ramsey numbers,
mixed Ramsey numbers,
multipartite Ramsey numbers,
online Ramsey numbers,
ordered Ramsey numbers,
ordered size Ramsey numbers,
oriented Ramsey numbers,
oriented size Ramsey numbers,
planar Ramsey numbers,
poset Ramsey numbers,
potential Ramsey numbers,
proper Ramsey numbers,
quasi-Ramsey numbers,
rainbow Ramsey numbers,
Ramsey equivalence,
Ramsey game numbers,
Ramsey games,
Ramsey multiplicities,
Ramsey-Turán numbers,
Ramsey sequences of graphs,
restricted online Ramsey numbers,
restricted size Ramsey numbers,
Schur numbers,
semi-algebraic Ramsey numbers,
set-coloring Ramsey numbers,
signed Ramsey numbers,
singular Ramsey numbers,
size Ramsey numbers,
size multipartite Ramsey numbers,
star-critical Ramsey numbers,
sub-Ramsey numbers,
Van der Waerden numbers,
weakened Ramsey numbers,
zero-sum Ramsey numbers,
or coloring graphs other than complete.

Interested readers can find such information in some of the surveys listed in Section 8 here.

Readers may be also interested in knowing that the US patent 6965854 B2 issued on November 15, 2005 claims a method of using Ramsey numbers in "Methods, Systems and Computer Program Products for Screening Simulated Traffic for Randomness." Check the original document at http://www.uspto.gov/patft if you wish to find out whether your usage of Ramsey numbers is covered by this patent.

9.2. Journal paper counts

Out of 965 references gathered in this survey, most are papers which appeared in more than 100 different periodicals (in addition to books, conference proceedings, arXiv postings, and personal communications). The most popular periodicals were:

Discrete Mathematics	101
Journal of Combinatorial Theory (old, Series A and B)	64
Journal of Graph Theory	63
Electronic Journal of Combinatorics	53
Graphs and Combinatorics	43
Ars Combinatoria	32
Journal of Combinatorial Mathematics and Combinatorial Computing	32
European Journal of Combinatorics	31
Discrete Applied Mathematics	24
Australasian Journal of Combinatorics	23
Combinatorica	21
Utilitas Mathematica	18
Combinatorics, Probability and Computing	16
SIAM Journal on Discrete Mathematics	16
Congressus Numerantium	12
Discussiones Mathematicae Graph Theory	11
Random Structures and Algorithms	9
Mathematica Applicata	8
Applied Mathematics Letters	7
arXiv preprints	35

Some of the original arXiv pointers remain listed together with their later published versions, if we think that this gives additional information.

The results of 192 references depend on computer algorithms.

Acknowledgements

In addition to the many individuals who helped to improve consecutive versions of this survey, the author would like to specially thank his Ramsey collaborator Xiaodong Xu. He has seen more revisions of this survey than one would wish. The author would also like to extend his thanks to Brendan McKay, Geoffrey Exoo and Heiko Harborth for their help in gathering data throughout the years. Those who contributed to the development and improvement of new revisions over the years are greatly appreciated.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections or additions will be greatly appreciated and considered for inclusion in the next revision of this survey.

References

The references are ordered alphabetically by the last name of the first author, and where multiple papers have the same first author they are ordered by the last name of the second author, etc. We preferred that all work by the same author be in consecutive positions. Unfortunately, this causes that some of the abbreviations are not in alphabetical order. For example, [BaRT] is earlier on the list than [BaLS]. We also wish to explain a possible confusion with respect to the order of parts and spelling of Chinese names. We put them without any abbreviations, often with the last name written first as is customary in original. Sometimes this is different from the citations in other sources. One can obtain all variations of writing any specific name by consulting the authors database of *Mathematical Reviews* at http://www.ams.org/mathscinet/search, or *zbMATH* (formerly *Zentralblatt für Mathematik*) at http://www.zbmath.org/authors.

Papers containing results obtained with the help of computer algorithms have been marked with stars. We identify two such categories of papers: those marked with * involving some use of computers where the results are easily verifiable with some computations, and those marked with ** where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special classes of graphs.

A

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E

[Ea1] Easy to obtain by simple combinatorics from other results, in particular by using graphs establishing lower bounds with smaller parameters.

[Ea2] Unique 2-(6,3,2) design gives lower bound 7, upper bound is easy.

[Ea3] Every edge (3, 3, 3;2)-coloring of K_{15} has 35 edges in each color [Hein], and since the number of triangles in K_{16} is not divisible by 3, hence no required triangle-coloring of K_{16} exists.

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