PDA → CFG Sipser p. 121

LEMMA 2.27

If a pushdown automaton recognizes some language, then it is context free.

PROOF IDEA

1. It has a single accept state, q_{accept}.
2. It empties its stack before accepting.
3. Each transition either pushes a symbol onto the stack (a *push* move) or pops one off the stack (a *pop* move), but it does not do both at the same time.

p. 122

PROOF Say that $P = (Q, \Sigma, \Gamma, \delta, q_0, \{q_{\text{accept}}\})$ and construct G. The variables of G are $\{A_{pq} \mid p, q \in Q\}$. The start variable is $A_{q_0, q_{\text{accept}}}$. Now we describe G's rules in three parts.

1. For each $p, q, r, s \in Q$, $u \in \Gamma$, and $a, b \in \Sigma_\varepsilon$, if $\delta(p, a, \varepsilon)$ contains (r, u) and $\delta(s, b, u)$ contains (q, ε), put the rule $A_{pq} \to a A_{rs} b$ in G.
2. For each $p, q, r \in Q$, put the rule $A_{pq} \to A_{pr} A_{rq}$ in G.
3. Finally, for each $p \in Q$, put the rule $A_{pp} \to \varepsilon$ in G.

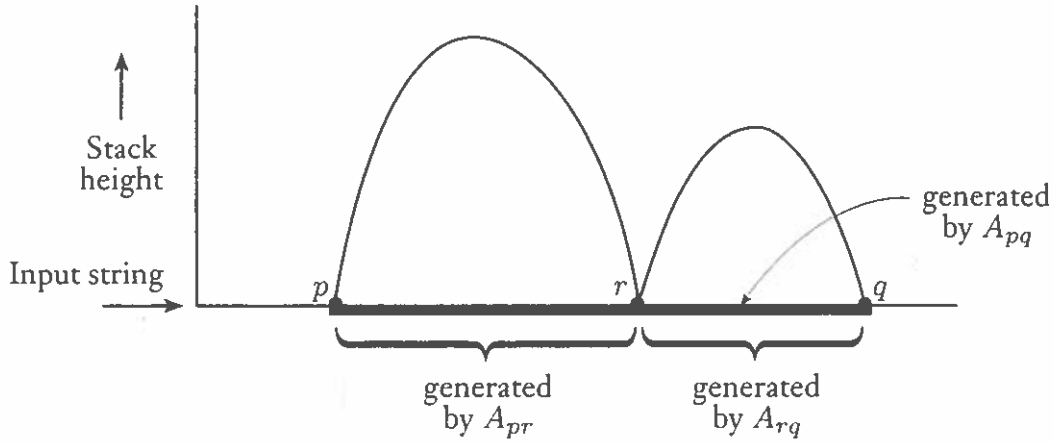

FIGURE 2.28
PDA computation corresponding to the rule $A_{pq} \rightarrow A_{pr}A_{rq}$

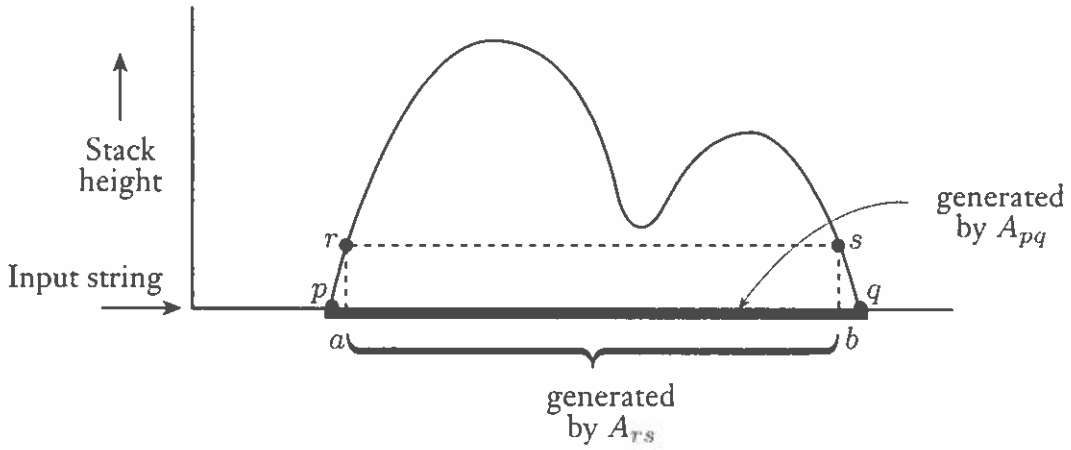

FIGURE 2.29
PDA computation corresponding to the rule $A_{pq} \rightarrow aA_{rs}b$

CLAIM 2.30

If A_{pq} generates x, then x can bring P from p with empty stack to q with empty stack.

CLAIM 2.31

If x can bring P from p with empty stack to q with empty stack, A_{pq} generates x.