Assignment 2
CSCI-661 Foundations of Computer Science Theory
Due Friday, February 9, 2024

- This homework is related to Chapters 0 and 1.1 of the textbook by Sipser.
- Each homework problem indicates the number of points it is worth for grading purposes. While individual problems may be worth more or fewer points than others, each homework assignment carries equal weight in the overall homework grade.

1. **(5 points)** Let A, B, and C be languages over some alphabet Σ. For each of the following statements, answer "yes" if the statement is **always** true, and "no" if the statement is not always true.

   (a) $A(B \cup C) \subseteq AB \cup AC$.

   (b) $A(B \cup C) \supseteq AB \cup AC$.

   (c) $A(B \cap C) \subseteq AB \cap AC$.

   (d) $A(B \cap C) \supseteq AB \cap AC$.

   (e) $A^* \cap B^* \subseteq (A \cap B)^*$.

   (f) $A^* \cap B^* \supseteq (A \cap B)^*$.

   (g) $A^*B^* \subseteq (AB)^*$.

   (h) $A^*B^* \supseteq (AB)^*$.

   (i) $A^* \supseteq (A^*)^*$.

   (j) $|A^*| > 0$.

2. **(5 points)** For each of the following languages, give two strings that are members and two strings that are not members – a total of four strings for each part. Assume the alphabet $\Sigma = \{a, b\}$ in all parts.

   (a) $\{a\}^*\{b\}^*$.

   (b) $\{a\}\{ba\}^*\{b\}$.

   (c) $\{a\}^* \cup \{b\}^*$.

   (d) $\{aaa\}^*$.

   (e) $\sum^*\{a\}\sum^*\{b\}\sum^*\{a\}$.

   (f) $\{aba\} \cup \{bab\}$.

(g) $(\{\epsilon\} \cup \{a\})\{b\}$.

(h) $(\{a\} \cup \{ba\} \cup \{bb\}) \sum^*$.

(i) $\{w \in \{a,b\}^* | n_a(w) = n_b(w)\}$.

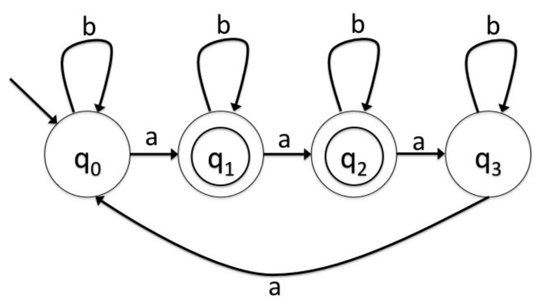
(j) $\{w \in \{a,b\}^* | \; |w| = 5\}$.

3. **(4 points)** Draw the state diagram of a finite automaton that accepts the language of all strings over $\{a,b\}$ that end with bb or contain the substring ba. Your finite automaton should not be overly complicated.

4. **(4 points)** Draw the state diagram of a finite automaton that accepts the language of all strings over $\{a,b\}$ that begin with aa and contain the substring aba. Your finite automaton should not be overly complicated.

5. **(4 points)** Draw the state diagram of a finite automaton that accepts the language of all strings over $\{a,b\}$ that do not contain an isolated pair of exactly two consecutive a symbols. For example, the strings aa, aab, baa, $babaab$ and $baabaaab$ are all rejected because they contain an isolated pair of exactly two consecutive a symbols. Strings such as ϵ, bb, a, aaa, $babab$ and $baaab$ are all accepted because any a symbols that appear in the string either appear in groups of one or consecutive groups of at least size three. Your finite automaton should not be overly complicated.

6. **(4 points)** Given the following state diagram for a DFA, M, specify the 5 components of the formal definition of $M = (Q, \Sigma, \delta, q_s, F)$.



7. **(4 points)** Apply the Cartesian product construction to construct a state diagram of a DFA for the following language:

$$L = \{w \in \{a, b\}^* \mid w \text{ has a even number of } b\text{'s and } w \text{ contains the substring } aa\}.$$

8. **(4 points)** Consider the following language:

$$L_1 = (\{0\} \cup \{1\})^*\{0\}\{1\}(\{0\} \cup \{1\})^*.$$

This language is the language of all strings over $\{0, 1\}$ that contain 01 as a substring. Notice that L_1 is expressed using the regular operations (union, concatenation, and Kleene star), and the languages $\{0\}$, $\{1\}$, $\{\epsilon\}$, and $\emptyset$.

(a) Let L_2 be the language of all strings over $\{0, 1\}$ wherein all occurrences of the 0 symbol occur in consecutive groups of two or more 0's.

Express L_2 using the regular operations and the languages $\{0\}$, $\{1\}$, $\{\epsilon\}$, and $\emptyset$.

(b) Let L_3 be the language of all strings over $\{0, 1\}$ that do not contain the substring 010.

Express L_3 using the regular operations and the languages $\{0\}$, $\{1\}$, $\{\epsilon\}$, and $\emptyset$.