Assignment 1
CSCI-661 Foundations of Computer Science Theory
due Tuesday, January 30, 2024

- This homework is related to Chapter 0 of the textbook by Sipser.
- Each homework problem indicates the number of points it is worth for grading purposes. While individual problems may be worth more or fewer points than others, each homework assignment carries equal weight in the overall homework grade.

1. **(6 points)** For each statement below, do the following:
   - Mathematically define the components of the statement. For example, let x represent a person, and let $P(x)$ represent a proposition about this person. Your proposition should not include the words "some" or "all" or "every", or similar language. If it does, you should introduce an additional variable to remove that language from the proposition.
   - Write the statement using your defined components and universal or existential quantifiers.
   - Write the negation of the statement mathematically. Carry the negation all the way in to the proposition.
   - Write in words what the negation of the statement is. Make sure your words match your mathematical negation. (Pay attention to where the negation occurs. Don't necessarily write words that are the most natural way to make the statement. Write words that match the mathematical representation.)

   (a) Some people like winter.
   (b) All people are happy every day.
   (c) Every triumph involves overcoming some obstacle.

2. **(5 points)** Let $S = \{1, 2, 3, 4\}$ and let $X = S \times S$. Define a relation R on X by:

$$(a, b)R(c, d) \quad \text{if and only if} \quad |a - b| = |c - d|.$$

   Show that R is an equivalence relation and list its equivalence classes. (Don't be thrown off by the fact that the elements of the set X happen to be ordered pairs.)

3. **(8 points)**

   (a) How many length 4 strings over $\{a, b\}$ are there? (You need not list them all.)

   (b) Let $k \geq 0$. How many length-k strings over $\{a, b\}$ are there? Simplify your answer as much as possible.

   (c) Let $k \geq 0$. How many length-k strings over $\{a, b\}$ that start with b are there? Simplify your answer as much as possible.

   (d) Let Σ be an alphabet with $|\Sigma| = d$, and let $k \geq 0$. How many length-k strings over Σ are there? Simplify your answer as much as possible.

   (e) Let Σ be an alphabet with $|\Sigma| = d$, and let $k \geq 0$. Prove by mathematical induction that the number of strings over Σ of length less than or equal to k is equal to $\frac{d^{k+1}-1}{d-1}$. Use your answer from (d) when formulating the problem.

4. **(5 points)** Prove by mathematical induction that

$$\sum_{i=1}^{n}(2i - 1) = n^2$$

   for all $n \geq 1$.

5. **(5 points)** Suppose you have access to an unlimited number of 3 gram and 5 gram weights.

   (a) What is the largest weight (integer number of grams) that can not be exactly represented using a combination of available weights? Just give the numerical answer, without proof.

   (b) Prove your assertion using mathematical induction. That is, prove that all values greater than k (where k is your answer from part (a)) can be represented as a combination of 3 gram and 5 gram weights. Hint: you may need multiple base cases for this problem.