Regularization in Neural Networks

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Regularization

- Bias / Variance Trade off [1]

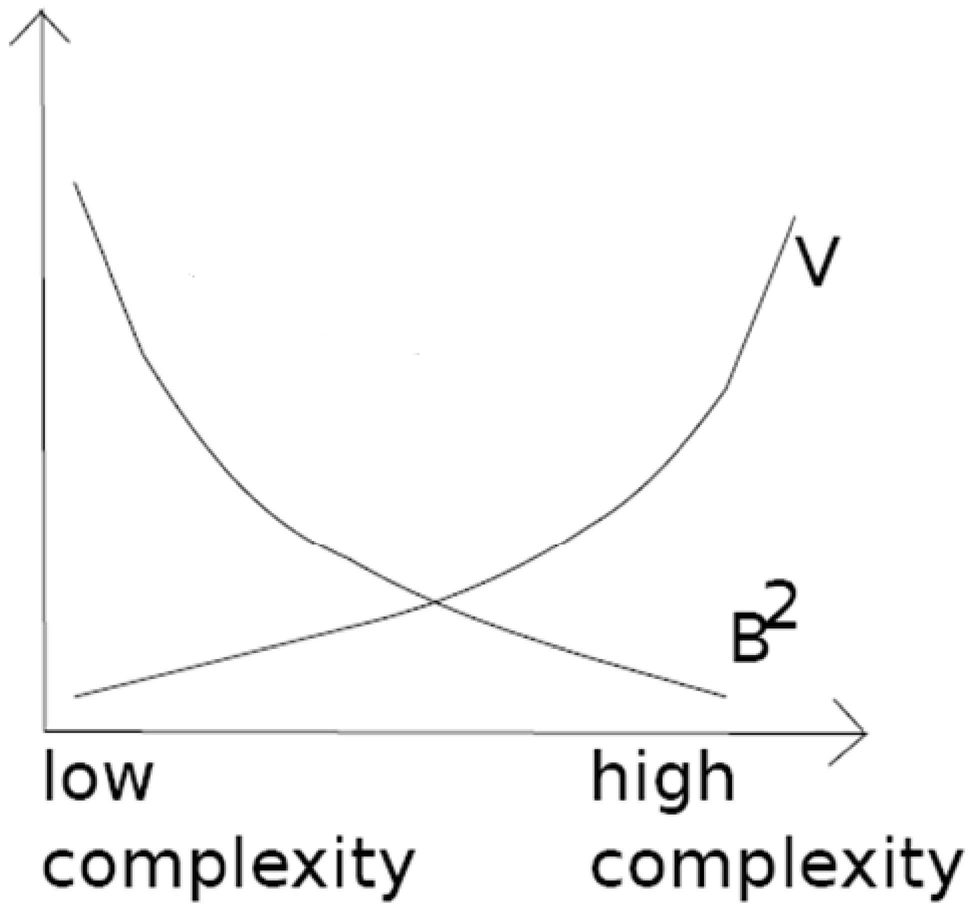
$$B(D) = E[g(z;D) - I(z)]$$

$$V(D) = E[(g(z;D) - E[g(z;D)])^2]$$

- Increasing bias will result in decreasing variance, and vice versa

[1] Justin, Domke, Lecture note of Statistical Machine Learning, 2009 Spring Quarter, RIT

Bias / Variance Trade off



[1] Justin, Domke, Lecture note of Statistical Machine Learning, 2009 Spring
Quarter, RIT

Bayesian Interpretation

- Given the prior density function of the weights $p(w)$
- The error function

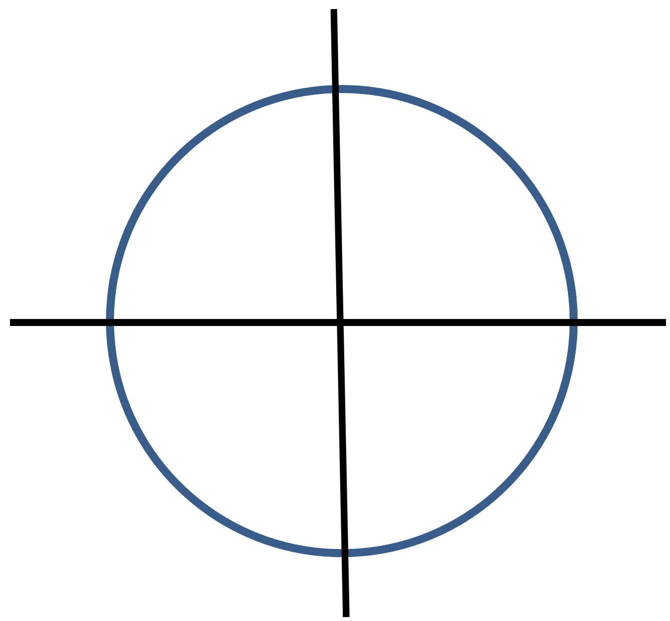
$$E = E(D) + \lambda E_w(w)$$

could be interpreted with conditional probability

$$P(w \mid D) \propto P(D \mid w) P(w)$$

Gaussian Weight Prior

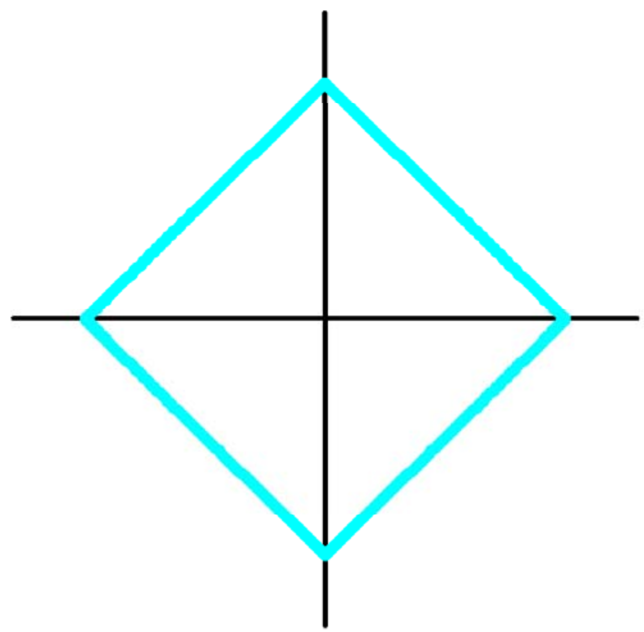
$$\tilde{E}(w) = E(w) + \lambda \|w\|^2$$



Laplacian Weight Prior

$$\tilde{E}(w) = E(w) + \lambda |w|$$

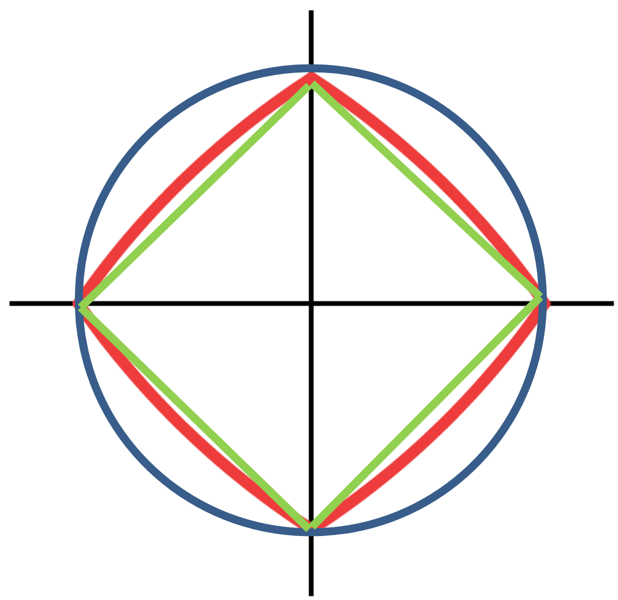
- Induces parameter sparsity by shrinking variables to exactly 0
- non-differentiable



Elastic Net Prior

$$\tilde{E}(w) = E(w) + \alpha(w^T w) + (1-\alpha)(|w|)$$

•Parameters go exactly to zero like In Laplacian priors but highly correlated variables are shrunk together like in Gaussian priors.



[2] H. Zou and T. Hastie, Regularization and Variable Selection via the Elastic Net, Royal Statistical Society Series B Statistical Methodology, 2005, VOL 67; No. 5, pp 768-768

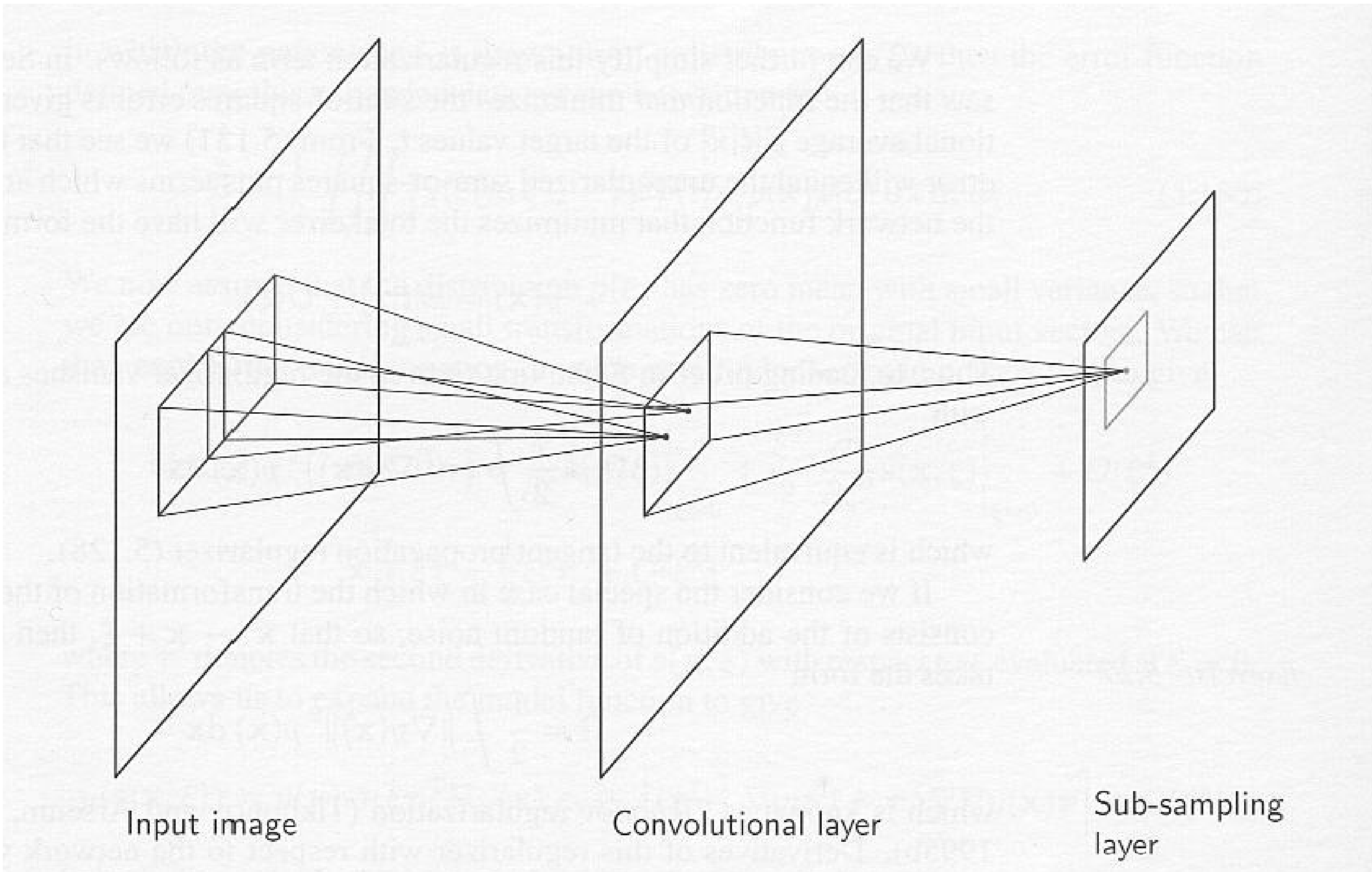
Early Stopping

- To end the training early before it converges
- Learning in NN is a highly non-convex optimization
- It is unclear exactly how early stopping will affect the objective function
- Under a quadratic error function early stopping is approximately a Gaussian prior

Invariances

- Regularization by making the classifier invariant to transformations of the data
- Can be done by adjusting the training data directly or the classifier directly

Convolutional Networks



Soft Weight Sharing

- The prior probability is a mixture of gaussians
- Pushes the weight values to form several groups
- The weights in the group tend towards the same value