Sample Midterm

Name: _____

- This exam consists of five questions.
- Each question is worth 10 points.
- The exam is worth 40 points.
- Your exam score is the sum of your four highest question scores.
- Calculators, cell phones, PDAs, etc. are not allowed.
- Use only scrap paper provided by me.
- Understanding the questions is part of the exam; you are not allowed to ask questions during the exam.
- As usual, you only have to explain your answer if this is stated in the question.
- You may only leave the room after you are done with the exam.

Note: This midterm was adapted from a midterm that used a different book. It has not been proofread as well as your midterm will be.

1. Let $\Sigma = \{0, 1\}$. In this question, every string over Σ is viewed as a binary natural number. Leading 0s are allowed. For example, 00100 is viewed as the number 4. ϵ is viewed as the number 0. Look at the following list of six languages.

   - $L_1 = \{x \in \Sigma^* \mid x \geq 16\}$.
   - $L_2 = \{x \in \Sigma^* \mid x \text{ is a square and } x < 1000\}$.
   - $L_3 = \{x \in \Sigma^* \mid x \text{ is not divisible by } 3\}$.
   - $L_4 = \{x \in \Sigma^* \mid x \text{ is divisible by } 666\}$.
   - $L_5 = \{xy \mid x, y \in \Sigma^* \text{ and } x * y = 12\}$.
   - $L_6 = \{xy \mid x, y \in \Sigma^*, |x| = |y|, \text{ and } x \text{ and } y \text{ represent the same number}\}$.

   (a) Which of the languages from the list are regular?

   (b) Which of the languages from the list contain ϵ?

   (c) Draw the transition diagram of a DFA that accepts one of the languages from the list *or* give a regular expression that represents one of the languages from the list. Also state which language you chose.

2. Write an algorithm that determines if two DFAs are equivalent. Your algorithm should take as input two DFAs M_1 and M_2 and should output "equivalent" if $L(M_1) = L(M_2)$ and "not equivalent" if they are not.

   You can use all constructions from the book and notes. Clearly state what you are using where.

3. Let $\Sigma = \{a, b\}$. Let *double* be the function from Σ^* to Σ^* that doubles each character in a string. For example, $double(baaba) = bbaaaabbaa$.

   (a) What is $double(aabaa)$?

   (b) What is $double(\epsilon)$?

   (c) Suppose $x \in \{a, b\}^*$ and the length of x is k. What is the length of $double(x)$?

   For L a language over Σ, define $double(L)$ as follows:

$$double(L) = \{double(x) \mid x \in L\}.$$

   (d) Let A be the language $\{ab, bbb, baba\}$. What is $double(A)$?

   (e) List all languages B over Σ that have the property that $double(B) = B$.

   (f) For each of the following statements, circle the right answer.
      i. If L is a regular language over Σ, then $double(L)$ is regular. True / False.
      ii. If L is a finite language over Σ, then $double(L)$ is finite. True / False.
      iii. If $L \subseteq \Sigma^*$ is not regular, then $double(L)$ is not regular. True / False.
      iv. If L_1 and L_2 are regular languages over Σ, then $L_1 \cup L_2$ is regular. True / False.
      v. If L_1 and L_2 are finite languages over Σ, then $L_1 \cup L_2$ is finite. True / False.
      vi. If $L_1 \subseteq \Sigma^*$ and $L_2 \subseteq \Sigma^*$ are not regular, then $L_1 \cup L_2$ is not regular. True / False.

   (g) For *one* of the six questions in part (f) where you answered false, give an explicit counter example. Also clearly state which question you chose.

4. Let $L = \{a^i b^k a^\ell \mid \ell > i + k\}$.

   (a) List all strings in L of length 7.

   (b) Use the Pumping Lemma for Regular Languages to prove that L is not regular.

5. This question is about the subset construction. In this question, take $\Sigma = \{a, b, c\}$.

   (a) If you literally apply the subset construction to an NFA with k states, how many states will the corresponding DFA have?

   (b) Give a simple example of a minimal NFA such that the DFA obtained by literally applying the subset construction is *not* minimal. (A minimal NFA is an NFA such that no equivalent NFA has fewer states.) For your answer, draw the transition diagram of the NFA, the transition diagram of the DFA obtained by applying the subset construction, and briefly argue that the DFA is not minimal.

   (c) Give a simple example of a minimal NFA such that the DFA obtained by literally applying the subset construction is minimal. For your answer, draw the transition diagram of the NFA, the transition diagram of the FA obtained by applying the subset construction, and briefly argue that the DFA is minimal.