Introduction to Computer Science Theory (4003-380-01)
Prof. Richard Zanibbi (20103, Spring 2011)
Homework 6, Due 4pm, Tuesday April 26, 2011

All questions are written for this assigment; submit them through myCourses or on paper before the start of class. If you work in a group of two, submit your answers only once, and make sure to include both your names on your submitted work.

Questions (50 points in total)

1. Show that Context-Free Languages are not closed under complement. In other words, show that for an arbitrary Context-Free Language L, $\overline{L}$ is not guaranteed to be a Context-Free Language.

2. Briefly describe the language defined by the CFG below, and then convert the CFG into an equivalent CFG in Chomsky Normal Form (CNF), using the procedure from Theorem 2.9.

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \epsilon \\ B &\rightarrow 00 \mid \epsilon \end{aligned}$$

3. Show that if G is a CFG in CNF that for any string $s \in L(G)$ with length $n \geq 1$, exactly $2n - 1$ steps are required in any derivation of s.

4. Show that the language of palindromes over {0,1} with an **equal number of 1's and 0's** is not context free.

5. Show that the language $F = \{a^i b^j \mid i = kj \ for \ k \geq 1\}$ is not context free.

6. Give parse trees and *leftmost* derivations for each of the following strings, according to the arithmetic expression grammar studied in class/provided in the textbook:

   (a) a x a
   (b) (a x (a + (a)))

7. Provide a context-free grammar for the complement of $L = \{0^i 1^i \mid i \geq 0\}$ (which *is* context-free).

8. Consider language $L = \{xcx^R cycy^R \mid x, y \in \{a, b\}^*\}$ (recall that x^R is the reversal of string x; c is a terminal symbol). Construct the PDA for L, and then convert it to a CFG using the construction provided in Lemma 2.27 of the course textbook.