

Introduction to Computer Science Theory (4003-380-01)
Prof. Richard Zanibbi (20103, Spring 2011)
Homework 1, Due 4pm, Tuesday March 15, 2011

You may submit your answers online through myCourses (as a .txt or .pdf file), or submit your homework on paper shortly before class starts (once lecture begins, the homework is late, and will not be graded). Handwritten answers are fine, but be aware that if an answer is unreadable, it will receive a low grade. Submitting (readable!) scans of handwritten work through myCourses is fine.

You may complete the homework in groups of two. **If you work in a team, submit only one paper/file, and make sure that both names appear on the submitted work.** Please see the course syllabus regarding rules and expectations for collaborating with others in the course.

Questions (20 points in total)

1. (10) For each of the following, indicate whether the statement is true (no proof or explanation is necessary), or false (in which case you must provide a counter-example). Counter-examples should be defined using the set-theoretic notation that we have been using to define languages in class. In the expressions below, parentheses (‘()’) are used to indicate the order in which language operations are applied.

- (a) $AB = BA$
- (b) $(AB)C = A(BC)$
- (c) $A(B \cup C) = AB \cup AC$
- (d) $A(B \cap C) = AB \cap AC$
- (e) $(A^R)^R = A$ (note that here we apply the reverse operation (R) to all strings in the argument (language))
- (f) $(AB)^R = A^R B^R$
- (g) $A^* \cup B^* = (A \cup B)^*$
- (h) $A^* \cap B^* = (A \cap B)^*$
- (i) $A^* B^* = (AB)^*$
- (j) $A^* = (A^*)^*$

2. (10) Consider the language $L \subseteq \{a, b\}^*$, which we define recursively as:

- (a) $a \in L$
- (b) For any string $x \in L$, $ax \in L$.
- (c) For any strings x and y in L , the strings $bx y$, xby and xyb are in L .
- (d) No other strings are in L .

Using structural induction, prove the following property for all strings in L :

$$\forall x \in L, N_a(x) > N_b(x)$$

In other words, that every string x in language L has more a's than b's.