A presentation slide titled "Logistics". To the left of the title is a decorative graphic consisting of overlapping colored squares (yellow, red, blue) and a black crosshair. A horizontal line extends from the graphic across the slide.

- Updates on assignments and such:
 - Assignment 2: 2D Algorithms
 - Most graded
 - Assignment 3: OpenGL 3D/Animation
 - Grading begun
 - Assignment 4: OpenGL Realism
 - Due Wednesday, November 7th
 - Midterm Pipeline Implementation
 - ALL GRADED
 - Homework 2: Transformations
 - ALL GRADED
 - Homework 3: Perspective
 - Past due / not graded
- All submissions via mycourses.



Logistics

- Final exam:
 - Wednesday, Nov 14
 - During class -- week 11
 - Brief review -- Monday Nov 12



Plan

- Small change in plans
 - This Week
 - 3D Pipeline assignment
 - Global Illumination
 - Next Week: Week 10
 - Modeling



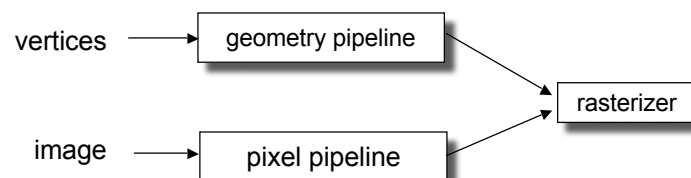
Goal for today's class

- Review of 3D pipeline
 - Complete understanding of the pipeline
 - Vertex
 - Rasterization
 - 3D Pipeline assignment.
 - OpenGL vs. simplified version



Geometry and Pixels

- Images and geometry flow through separate pipelines that join at the rasterizer
- Advantage: visual detail is in the image, not the geometry
 - "Complex" textures do not affect geometric complexity





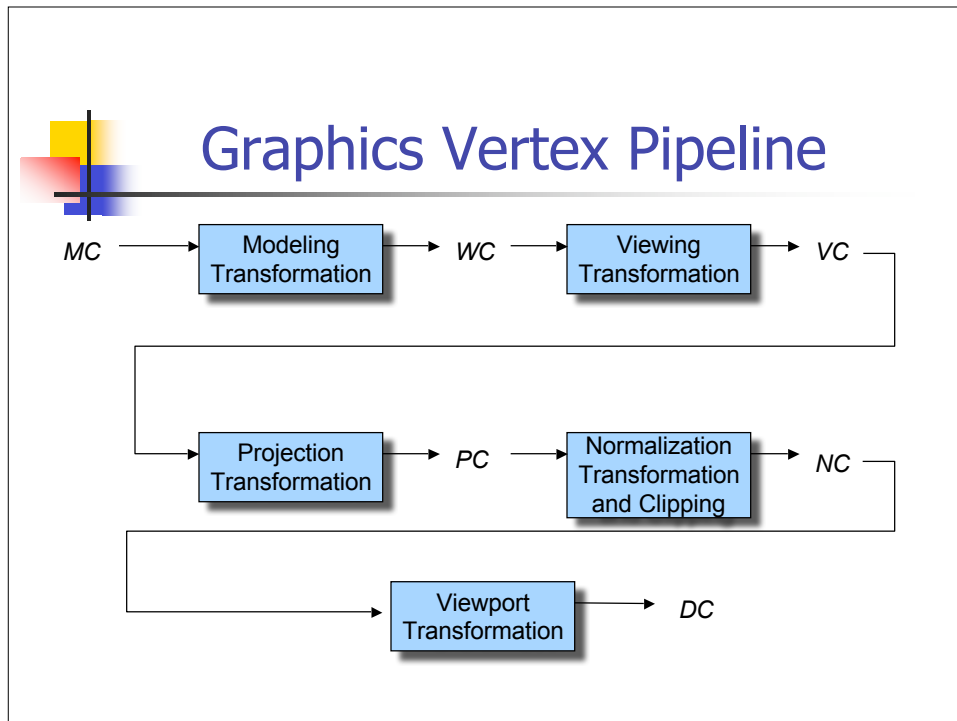
Vertex Pipeline

- Operates on polygon vertices
 - Transform, transform, transform
 - The composite matrix.
 - Eventually ends in transformation to 2D device space (integer -- discrete)



The composite matrix

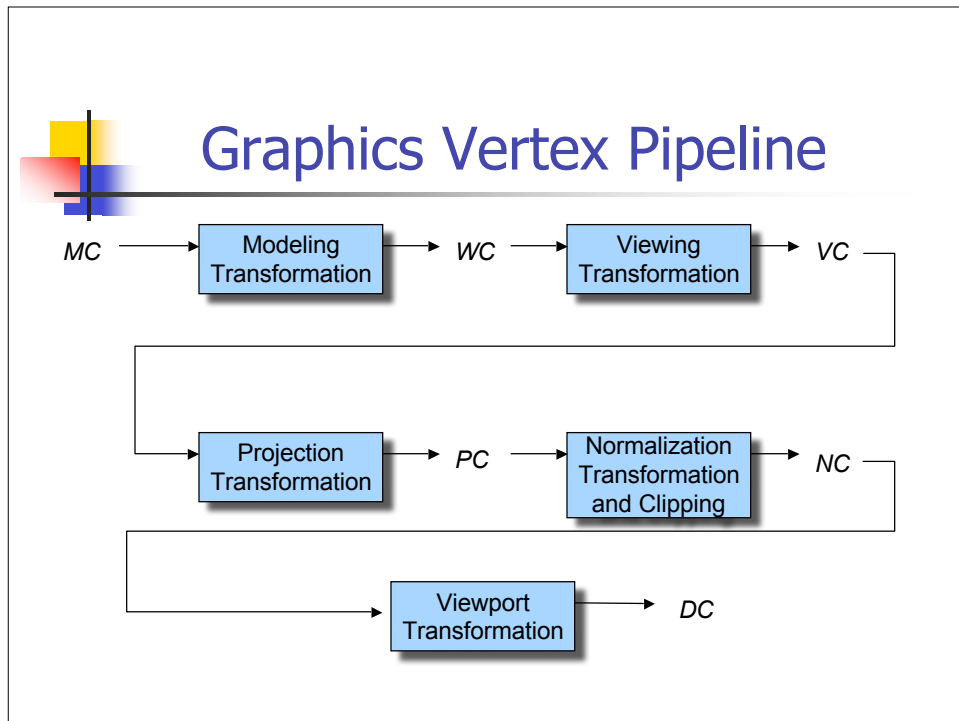
- Single 4x4 Matrix that will take you from model to device.
- In OpenGL
 - Set up before defining polygon vertices
 - Each vertex will get transformed by "current" composite.



Model Coordinates

- Coordinates system in which the polygon is defined.

```
glBegin(GL_TRIANGLES);  
    glVertex3f(-1.0f, -0.5f, -4.0f);  
    glVertex3f( 1.0f, -0.5f, -4.0f);  
    glVertex3f( 0.0f,  0.5f, -4.0f);  
glEnd();
```

World Coordinates

- Coordinate system of your world.
 - Transformations: Translation, Rotation, Scaling.

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

translation scaling Rotate x Rotate y Rotate z



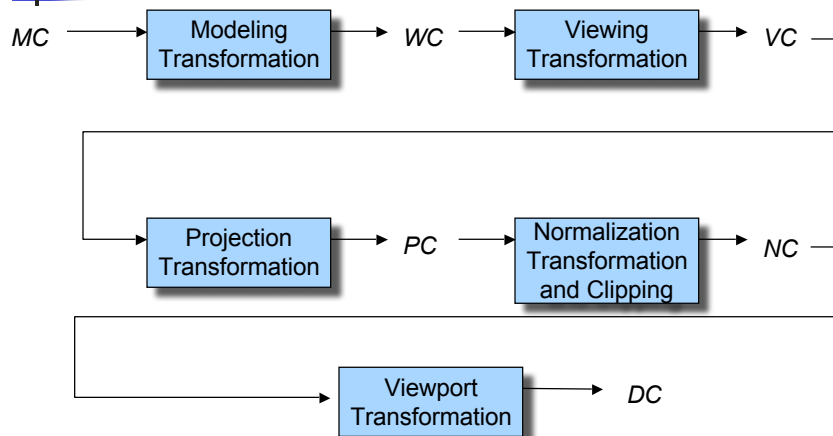
World Coordinates

- Defining transformations

```
glTranslatef( tx, ty, tz )  
glScalef( sx, sy, sz )  
glRotatef( angle, x, y, z )
```



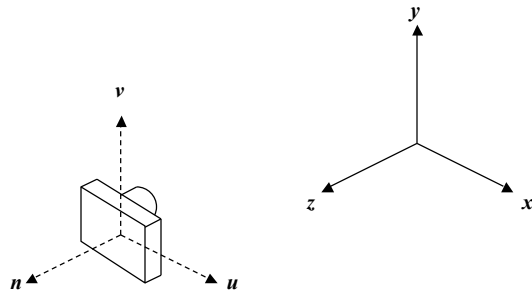
Graphics Vertex Pipeline





View Coordinates

- Sometime called camera coordinates
- World as seen from pov of camera



View transform

- The *eyepoint*, *lookat*, and the *up* vector define the *viewing-reference coordinate* system, also known as camera or eye coordinates.
- The default camera orientation has the camera/viewing-reference/eye coordinate system **coincident** with the world axes, i.e.,
 - *Eyepoint* at $(0, 0, 0)$
 - *Lookat* $(0, 0, -1)$
 - *Up* vector $(0, 1, 0)$ anchored at *eyepoint*
 - Then
 - $n = \text{eyepoint} - \text{lookat}$ is $(0, 0, 1)$ (normalized*)
 - $u = \text{up} \times n$ is $(1, 0, 0)$ (normalized)
 - $v = n \times u$ is $(0, 1, 0)$
 - *normalized means a vector of length 1

```
gluLookAt( eye.x, eye.y, eye.z, lookat.x,
            lookat.y, lookat.z, up.x, up.y, up.z )
```



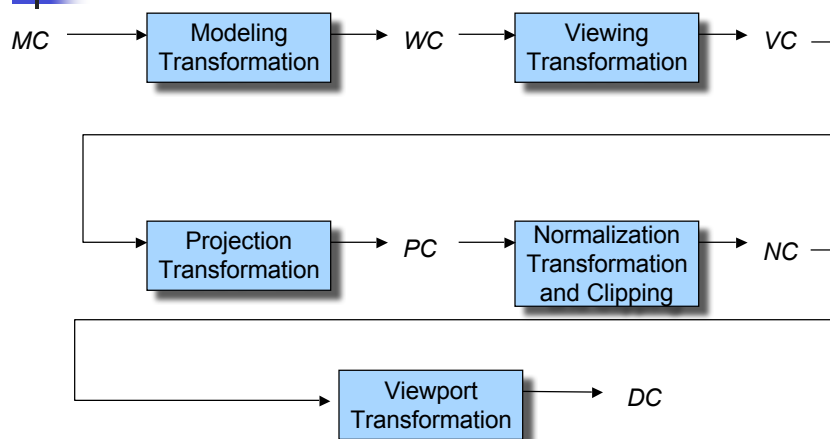

World to Camera Coordinate Transformation

- So, we can do something similar in 3D
 - Build your own set of world and camera coordinates
 - Come up with the four transformations needed to make the axes coincide
 - These are the same four transformations that will be applied to all vertices in the world to get them in terms of camera coordinates
 - The composite of these is equivalent to

$$\begin{pmatrix} u_x & u_y & u_z & -\mathbf{u} \cdot \text{eyepoint} \\ v_x & v_y & v_z & -\mathbf{v} \cdot \text{eyepoint} \\ n_x & n_y & n_z & -\mathbf{n} \cdot \text{eyepoint} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Graphics Vertex Pipeline



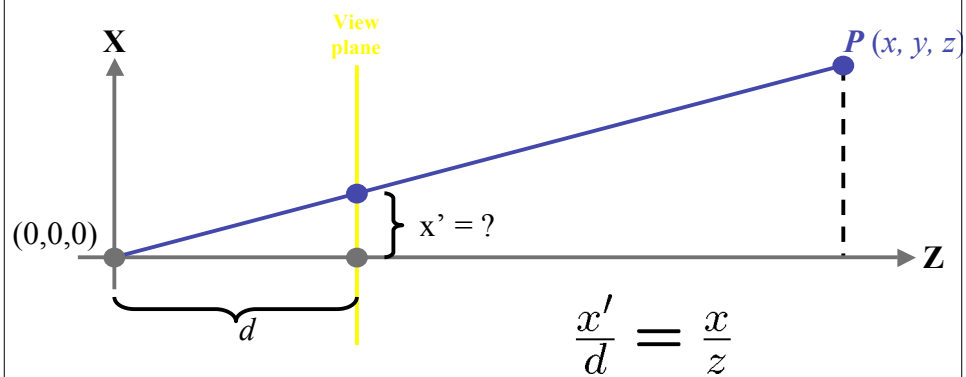


Projection

- Projecting 3D points onto “film plane”
 - Orthographic
 - Perspective



Perspective Projection





Perspective Projection

$$M_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

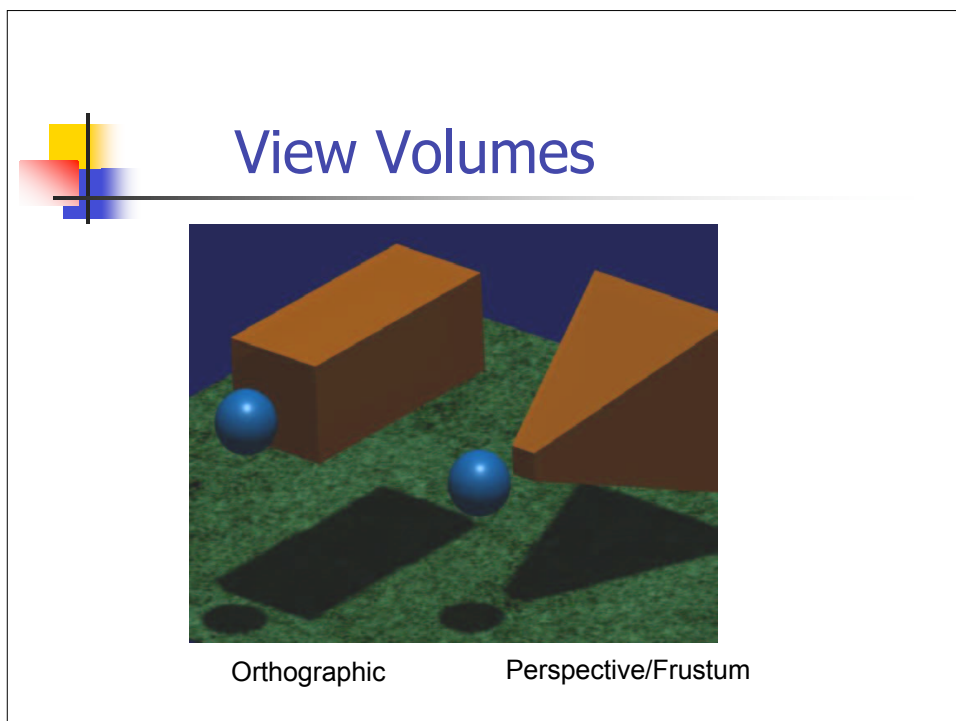
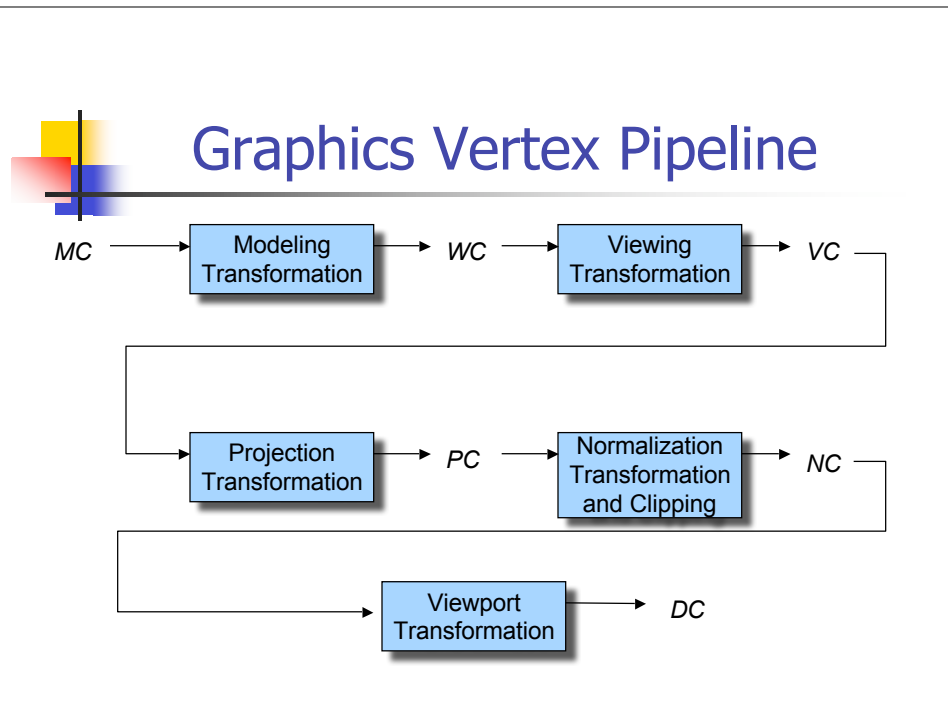
`glFrustum(left, right, bottom, top, near, far)`



Orthographic Projection

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

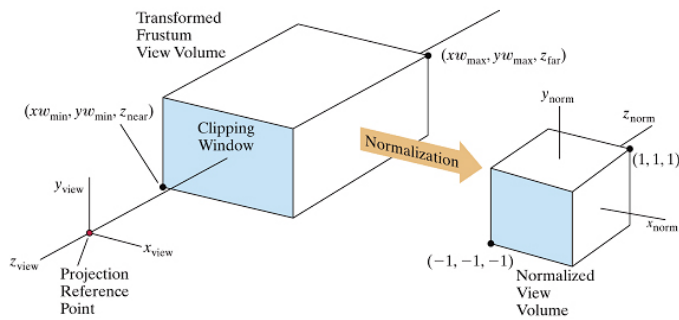
`glOrtho(left, right, bottom, top, near, far)`





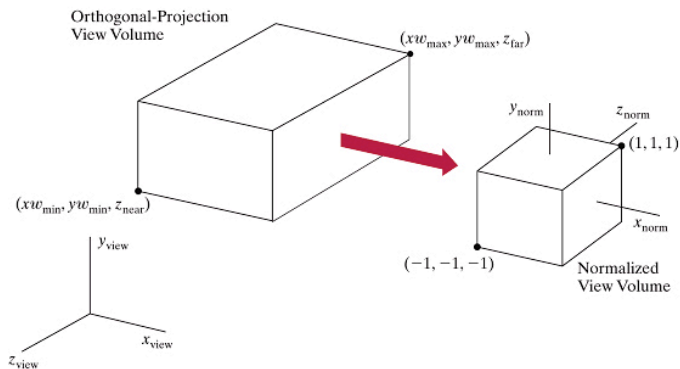
Frustum Normalization in OpenGL

Remember that all projections in OpenGL map the view volume to a 2 X 2 X 2 cube.



Orthographic Normalization in OpenGL

All projects in OpenGL map the view volume to a 2 X 2 X 2 cube.





Normalization

- Converts points in view volume to a standard canonical space.



Normalization - Ortho

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & \frac{-(right+left)}{right-left} \\ 0 & \frac{2}{bottom-top} & 0 & \frac{-(bottom+top)}{bottom-top} \\ 0 & 0 & \frac{2}{far-near} & \frac{-(far+near)}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

`glOrtho(left, right, bottom, top, near, far)`



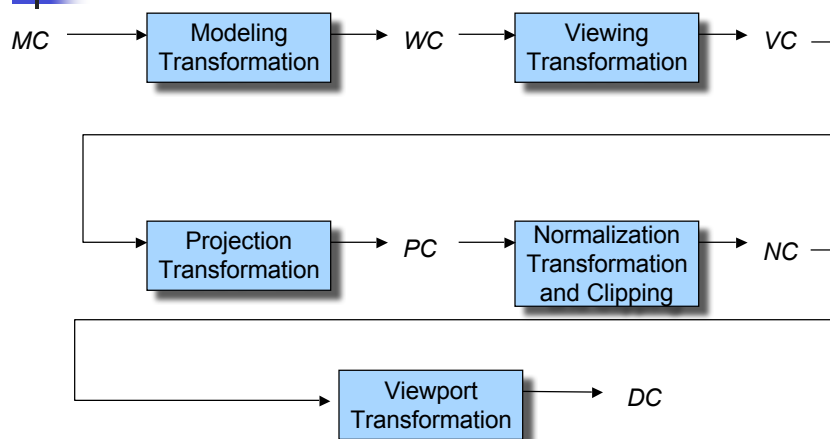
Normalization - Perspective

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2 \times \text{near}}{\text{right} - \text{left}} & 0 & \frac{-(\text{right} + \text{left})}{\text{right} - \text{left}} & 0 \\ 0 & \frac{2 \times \text{near}}{\text{bottom} - \text{top}} & \frac{-(\text{bottom} + \text{top})}{\text{bottom} - \text{top}} & 0 \\ 0 & 0 & \frac{\text{far} + \text{near}}{\text{far} - \text{near}} & \frac{-2 \times \text{far} \times \text{near}}{\text{far} - \text{near}} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

`glFrustum(left, right, bottom, top, near, far)`



Graphics Vertex Pipeline





Device (Screen) coords

- Viewport transform
 - Transforms points from canonical space to points on screen



Viewport Transform

- The normalized device coordinates (x, y) ,
 - $-1 \leq x \leq +1$ and $-1 \leq y \leq +1$, are converted into pixel coordinates $\text{left} \leq x_w \leq \text{left} + \text{width}$ and $\text{bottom} \leq y_w \leq \text{bottom} + \text{height}$.
 - The z coordinate $-1 \leq z \leq +1$ is scaled in the range $0 \leq z_w \leq 1$.



Viewport Transformation

- The sequence:

$$T(u_{min}, v_{min}) \cdot S(s_x, s_y) \cdot T(-x_{min}, -y_{min})$$

- The matrices:

$$= \begin{bmatrix} 1 & 0 & x_{v_{min}} \\ 0 & 1 & y_{v_{min}} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{x_{v_{max}} - x_{v_{min}}}{x_{w_{max}} - x_{w_{min}}} & 0 & 0 \\ 0 & \frac{y_{v_{max}} - y_{v_{min}}}{y_{w_{max}} - y_{w_{min}}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_{w_{min}} \\ 0 & 1 & -y_{w_{min}} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x_{v_{max}} - x_{v_{min}}}{x_{w_{max}} - x_{w_{min}}} & 0 & -x_{w_{min}} \times \frac{x_{v_{max}} - x_{v_{min}}}{x_{w_{max}} - x_{w_{min}}} + x_{v_{min}} \\ 0 & \frac{y_{v_{max}} - y_{v_{min}}}{y_{w_{max}} - y_{w_{min}}} & -y_{w_{min}} \times \frac{y_{v_{max}} - y_{v_{min}}}{y_{w_{max}} - y_{w_{min}}} + y_{v_{min}} \\ 0 & 0 & 1 \end{bmatrix}$$



Viewport Transform

$$x_w = left + width \cdot \frac{x + 1}{2}$$

$$y_w = bottom + height \cdot \frac{y + 1}{2}$$

$$z_w = \frac{z + 1}{2}$$

`glViewport(left, bottom, width, height)`



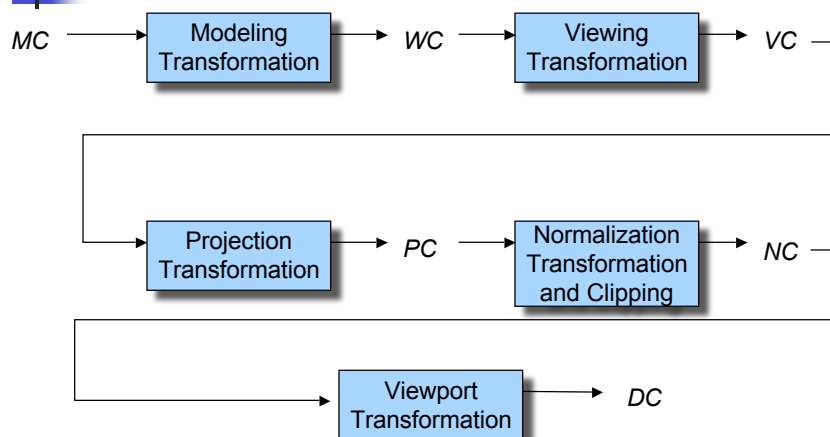
Viewport Transformation

- Note that this is a 3D matrix
 - To get from 4D to 3D, perform the homogeneous transform:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow \begin{bmatrix} x/w \\ y/w \\ z/w \end{bmatrix}$$



Graphics Vertex Pipeline



Watch the order in which matrices are applied



At this point...

- You still only have vertices
- All vertices have been converted to screen coordinates.
- You are ready for rasterization.

- Questions?



Rasterization

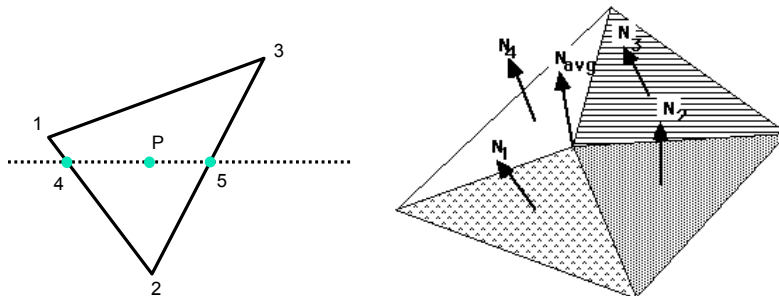
- Convert vertices to polygons
- Issues:
 - Need to fill interior points
 - What color to shade?
 - Which polygon to draw at a given pixel?
- We will modify the basic scan line algorithm.
- But first, we'll need more info at each of the vertices.

Shading Models

- Gouraud Shading
 - Illumination is interpolated across each polygon
 - Normals required at each polygon vertex, calculated as an average of the normals of each face that shares that vertex
 - Illumination is calculated for each polygon vertex
 - Interior points interpolated from endpoint illumination intensities

Shading Models

- Gouraud Shading – interpolating normals





Shading Model

- Gouraud Shading

Need to maintain
color at each vertex



Hwang



Calculating color

- Lighting
- Material properties
- Textures
- More info to maintain at each vertex
 - Material properties
 - Normals
 - Texture coords



Which polygon to render

- From Wikipedia:
 - *hidden surface determination* is the process used to determine *which surfaces and parts of surfaces* are *not visible* from a *certain viewpoint*.
 - A hidden surface determination algorithm is a solution to the *visibility problem*
 - the core differences between most rendering algorithms is how they handle this problem.



Stages of Hidden Surface Determinations

- Viewing Fustrum Culling
 - Remove objects not in view volume
 - I.e Clipping
- Backface culling
 - Remove faces that do not face the camera
- Occlusion Culling
 - Determines which portions of objects are hidden by other objects from a given viewpoint.
- Contribution Culling
 - Objects whose screen projection are so small are thrown away.



Depth (Z) Buffer Algorithm

- Requires two arrays the size of the screen
 - Depth – initialized as “far away”
 - Color – initialized to background color
- Used in OpenGL

```
For each polygon,  
  For each y,  
    For each x ,  
      If  $z < \text{depth}[x][y]$ ,  
        Depth[x][y] = z  
        Color[x][y] = polygon color
```



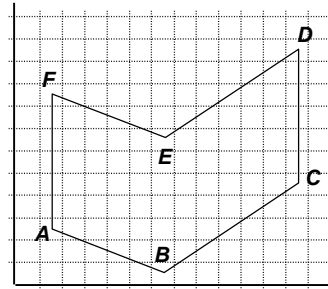
Basic scan line algorithm

- Must do this for all edges and all scan lines, not just one edge
- Can generalize this to handle arbitrarily complex polygons using an *edge table*
- The ET contains all edges, sorted by y_{min}
- We move edges from ET into an *active edge table* as we encounter them
- Note: our text calls AET the active edge list (AEL)



Building the Edge Table

- Our figure's edges:
 - AB, BC, CD, DE, EF, FA
- We create a bucket list for each scan line which indicates which edge(s) intersect that scan line
- Each edge is listed only the *first time* it is crossed
 - E.g., FA will appear for scan line 3, but not for lines 4-9
- Bucket chains are sorted by increasing x of the lower endpoint
 - What if multiple buckets have the same x endpoint?

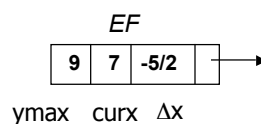


$A = (2,3)$ $B = (7,1)$
 $C = (13,5)$ $D = (13,11)$
 $E = (7,7)$ $F = (2,9)$

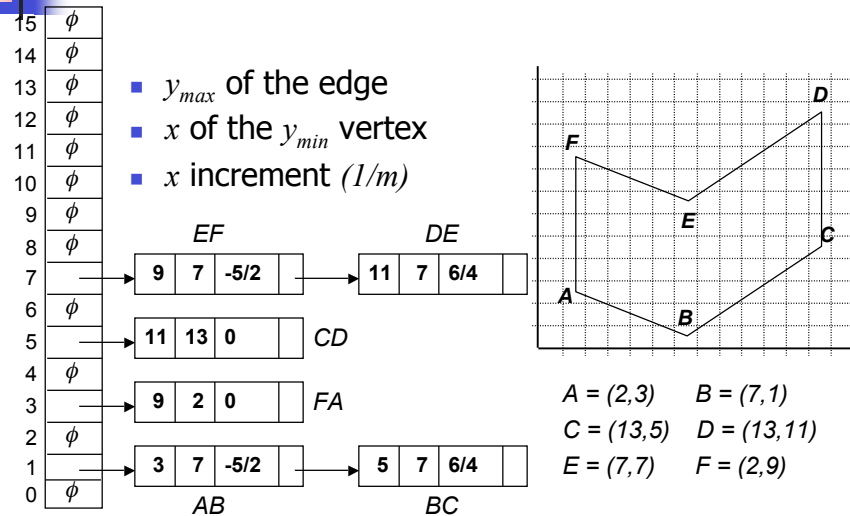


Edge Table Buckets

- Each bucket contains the following things:
 - y_{max} of the edge
 - Current x of the edge
 - Initially, x of the vertex with the y_{min} coordinate
 - The x increment ($1/m$)
 - A link to the next bucket
- We build the ET as an array of pointers to buckets, indexed by scan line number



Edge Table Example



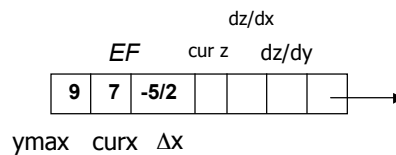
Fill algorithm

- Form the ET
- Initialize AET to empty
- Set y to the smallest y in the ET which has buckets
- Repeat until both ET and AET are empty:
 - Remove AET entries where $y = y_{max}$
 - Move from $ET[y]$ to AET when $y_{min} = y$
 - Sort AET on x
 - Fill pixels on scan line y using pairs of x coords from AET
 - Increment y
 - For each non-vertical edge in AET, update x for new y
 - How to detect non-vertical edge?
 - Look at inverse slope - if $1/m \neq 0$, have non-vertical edge



Adding to the basic scan line algorithm

- Add other values to be interpolated to the edge table buckets
 - E.g.
 - Z (depth) values
 - Colors
 - Texture coords



Questions?



The FINAL Option

- For the daring...
 - Build a 3D pipeline implementing the vertex and rasterization pipeline presented today.



The FINAL Option

- Vertex additions
 - 3D transforms
 - Projections (Ortho and Frustrum)
 - 3D Camera Model
 - Perspective divide
 - Storage of depth and color at vertices
- CLIPPING in 2D



The FINAL Option

- Rasterization Additions
 - Basic Z-Buffer algorithm
 - Z value and color interpolation in scan line algorithm.
 - Gouraud Shading
- NO PHONG Illumination
- NO TEXTURE Mapping



The FINAL Option

- Your own implementation of select OpenGL / Glu routines.
- Use modified 2D drawing routines from Midterm / assignment 2
- Only allowed to use setPixel().



The FINAL Option

- List of functions to implement with complete write-up (and test program) by Wednesday.
- Due date:
 - Wednesday, November 14th.
- Questions?