CSCI 740 - Programming Language Theory

Lecture 33

Automata Theoretic Model Checking

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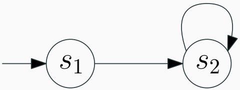
Recap

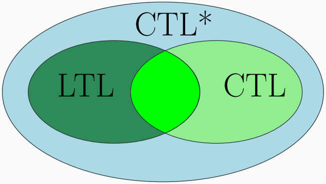
Model

- Finite-state Machine
- Infinite runs



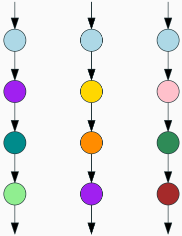
Specification

- Linear Time Logic (LTL)
- Computation Tree Logic (CTL)
- CTL*



LTL

- Unique successors
- Infinite runs (words)
- Operators:
  - Finally F
  - Globally G
  - Next X
  - Until U



LTL Model Checking Problem

- LTL model checking seeks to answer the question

$$\text{Does } M \models \phi \text{ hold?}$$

- or, equivalently:

$$\text{Does } \forall \phi \in \mathsf{Path}(M).\ \pi_0 \models \phi \text{ hold?}$$

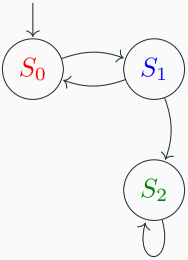
- The universal quantification is over the infinite set of paths, and each path is infinitely long
- How can we check infinitely many paths?

Language of Transition System

- For a Kripke structure $M = \langle S, S_0, R, L \rangle$
- Let's consider the set of states S as an alphabet Σ
- Each infinite path π is then an infinite word
- The set of all paths of M is the language $L(M)$ accepted by M

Example.

M



$L(M)$

$$\{S_0S_1S_2S_2S_2\cdots$$
$$S_0S_1S_0S_1S_2S_2S_2\cdots$$
$$S_0S_1S_0S_1S_0S_1S_2S_2S_2\cdots$$
$$S_0S_1S_0S_1S_0S_1S_0S_1S_2S_2S_2\cdots$$
$$\cdots,$$
$$S_0S_1S_0S_1S_0S_1S_0S_1S_0S_1S_0S_1\cdots$$
$$\}$$

Automata over Finite Traces

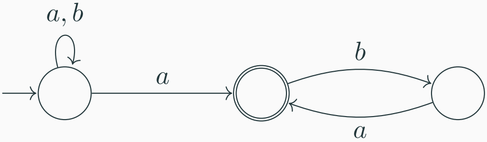
- (Regular) Finite automaton with accepting states
- All finite traces (words) that take the automaton into the accepting state are “in its language”
- But behaviors (and traces) have infinite length
- So we need a new notion of acceptance

Automata over Infinite Traces

- ω-automata: accepts (or reject) words of infinite length
- An ω-word is an infinite sequence of letters
- The set of all ω-words is denoted by Σ^ω
- There are different kinds of ω-automata
  - Büchi automata, Rabin automata, Street automata, parity automata, ...
- (non-deterministic) **Büchi** automata are commonly used for model checking
- They express the ω-regular languages

Büchi Automata

- Same syntax as DFAs and NFAs, but different acceptance condition
- A run of a Büchi automaton on an ω-word is an infinite sequence of states and transitions
- A run is accepting if it visits the set of final states infinitely often
- Final states renamed to accepting states



Language of DFA (regular expression):

$$(a + b) * (ab)*$$

If interpret as a Büchi automaton over infinite words: (ω-regular expression):

$$(a + b) * (ab)^\omega$$

Büchi Automata

A (non-deterministic) Büchi automaton $\langle S, \Sigma, \rightarrow, S_0, A \rangle$ consists of:

- S a finite set of states
- Σ an alphabet
- $\rightarrow \subseteq S \times \Sigma \times S$ transition relation
- $S_0 \subseteq S$ set of initial states
- $A \subseteq S$ set of accepting states

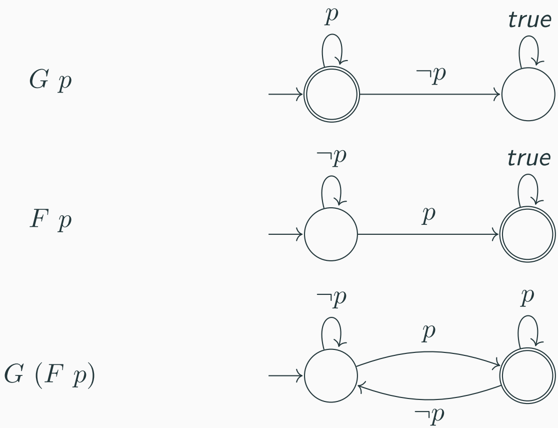
An infinite word is accepted by a Büchi automaton iff there is a run of the automaton on which some accepting state is visited infinitely often

Exercise

Construct Büchi automata accepting the following languages over $\Sigma = \{a, b, c\}$

- $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains ab exactly once$\}$
- $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains ab at least once$\}$
- $L_3 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains ab infinitely often$\}$
- $L_4 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains ab only finitely often$\}$

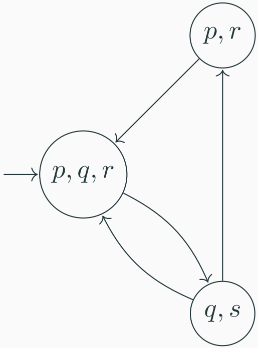
From LTL Properties to Büchi Automata

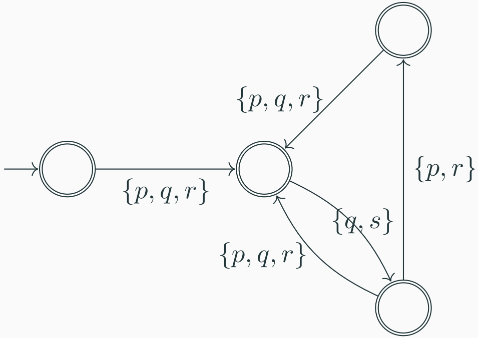


- Size of the property automaton can be exponential in the size of the LTL formula
  - Recall the complexity of LTL model checking

From Kripke to Büchi

- Introduce a new initial state
- Move the labels on a state to the incoming edge(s)
- Make all states final





Combining Büchi Automata

Büchi automata are closed under

- Union (similar to the case of finite automata)
- Intersection
- Complement

Büchi Automata: Language Emptiness Check

- Given a Büchi automaton, is the language accepted by the automaton empty?
- i.e., does it accept any string?
- A Büchi automaton accepts a string when the corresponding run visits an accepting state infinitely often

To check emptiness:

- Look for a cycle which contains an accepting state and is reachable from the initial state
- Find a strongly connected component that contains an accepting state, and is reachable from the initial state
- If no such cycle can be found the language accepted by the automaton is empty

LTL Model Checking Idea

We reformulate the LTL model checking problem to:

$$L(M) \cap \overline{L(\phi)} = \emptyset$$

Now:

1. Observe that $\overline{L(\phi)} = L(\neg\phi)$
2. Let A_ϕ be a Büchi automaton such that $L(\phi) = L(A_\phi)$
3. Compute the product of M and A

$$L(M \times A) = L(M) \cap L(A)$$

4. So, to check $M \models \phi$, instead check

$$L(M \times A_{\neg\phi}) = \emptyset$$