

Garbage Collection

1 Introduction

You will undertake a few simple problems in order to analyze the performance of garbage collection.

2 Description

- (Adapted from Exercise 6 of Chapter 4 from *Programming Languages: Build, Prove, and Compare* (p. 342).)

Derive a formula to express the cost of mark-and-sweep garbage collection as a function of γ . The cost should be measured in units of *GC work per allocation*. For a very simple approximation, assume a fixed percentage of whatever is allocated becomes garbage by the next collection.

You should not find it necessary to assume that you know anything else about the heap besides γ .

Consider the mark-and-sweep garbage collection described in the textbook that uses lazy unmarking.

Hint: Consider the execution of the program (mutator, allocator, and garbage collector) starting immediately after the conclusion of a garbage collection that results in a heap satisfying $H = \gamma \times L$ (where L measures the number of live objects and all objects are of the same size) and ending immediately after the conclusion of the next garbage collection that also results in heap satisfying $H = \gamma \times L$ (although a different set of L objects may be live).

- (Adapted from Exercise 18 of Chapter 4 from *Programming Languages: Build, Prove, and Compare* (p. 344).)

Derive a formula to express the cost of copying garbage collection as a function of γ . The cost should be measured in units of *GC work per allocation*. The units of work may be different from the units used in the previous problem. For a very simple approximation, assume a fixed percentage of whatever is allocated becomes garbage by the next collection.

You should not find it necessary to assume that you know anything else about the heap besides γ .

Hint: Consider the execution of the program (mutator, allocator, and garbage collector) starting immediately after the conclusion of a garbage collection that results in a heap satisfying $H = \gamma \times L$ (where L measures the number of live objects and all objects are of the same size) and ending immediately after the conclusion of the next garbage collection that also results in heap satisfying $H = \gamma \times L$ (although a different set of L objects may be live).

- (Adapted from Exercise 9 of Chapter 4 from *Programming Languages: Build, Prove, and Compare* (p. 342).)

The mark-and-sweep allocator skips (and unmarks) marked objects until it finds an unmarked object. Assume that the marked objects are distributed independently, so that the probability of any particular object being marked is $\frac{1}{\gamma}$.

1. Derive a formula giving, as a function of γ the probability that the allocator skips exactly zero objects.
2. Derive a formula giving, as a function of γ the probability that the allocator skips exactly one object.
3. Derive a formula giving, as a function of γ the probability that the allocator skips exactly two objects.
4. Derive a formula giving, as a function of γ the probability that the allocator skips exactly k objects.
5. Derive a formula giving, as a function of γ the probability that the allocator skips at most three objects. Calculate this number for some interesting values of γ , e.g., $\gamma \in \{1, 1.1, 1.2, 1.5, 2, 3, 5\}$.
6. Derive a formula giving, as a function of γ , the expected number of marked objects that the allocator must skip (and unmark) before finding an unmarked object. Calculate this number for some interesting values of γ .

Hint: “the expected value of a discrete random variable is the probability-weighted average of all possible values. In other words, each possible value the random variable can assume is multiplied by its probability of occurring, and the resulting products are summed to produce the expected value.”
http://en.wikipedia.org/wiki/Expected_value

Hint: $\sum_{k=0}^{\infty} k \times r^k = \frac{r}{(1-r)^2}$ if $r < 1$.

7. Is the assumption that the marked objects are distributed independently justified?

3 Requirements and Submission

At the end of class, submit the group’s solutions as hard-copy; be sure to include the names of all group members in the submission.