CSCI 742 - Compiler Construction

Lecture 7
Building Efficient Lexers
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January 31, 2018

Lexer Automatic Construction: Big Picture

Input: Token Spec

- List of regular expressions (RE) in priority order

Output: Lexer

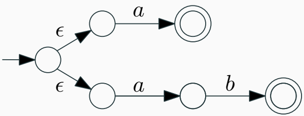
- Reads an input stream and breaks it up into tokens according to REs

Algorithm

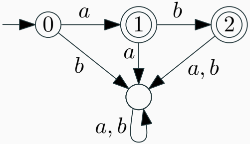
- Convert REs into non-deterministic finite automata (NFA)
- Convert NFA to DFA
- Convert DFA into transition table

Lexer Automatic Construction: Example

- RE for tokens: $(a|ab)$
- NFA:



- DFA:



- Transition Table:

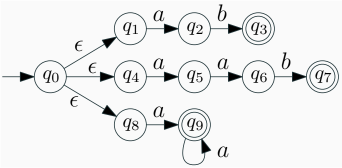
	a	b
0	1	Error
1	Error	2
2	Error	Error

Lexer Automatic Construction: Example

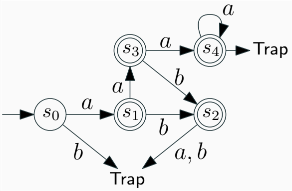
Token Specification

ab	{Action 1}
aab	{Action 2}
$a+$	{Action 3}

NFA



DFA



	a	b
s_0	s_1	Error
s_1	s_3	s_2
s_2	Error	Error
s_3	s_4	s_2
s_4	s_4	Error

$$\Sigma = \{a, b\}$$

Example:

Input: `aab`

- $s_0 \longrightarrow s_1 \longrightarrow s_3 \longrightarrow s_2$

Kleene's Theorem

Theorem

A language L can be described by regular expression if and only if L is the language accepted by a finite automaton.

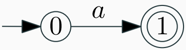
Algorithms:

- Regular expression $\Rightarrow$ Automaton
  - important for lexer construction
- Automaton $\Rightarrow$ Regular expression
  - interesting method in formal languages theory

RE $\Rightarrow$ Finite Automaton

- Build the finite automaton inductively, based on the definition of regular expressions

a



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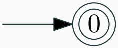
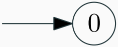
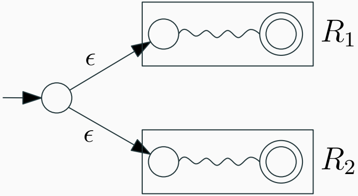


$\emptyset$

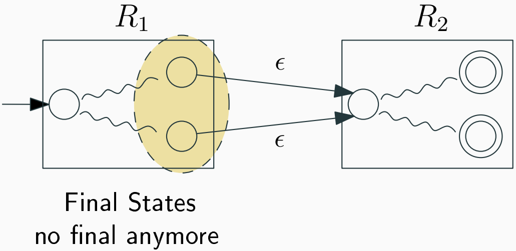


RE $\Rightarrow$ Finite Automaton

Alternation $R_1 \mid R_2$

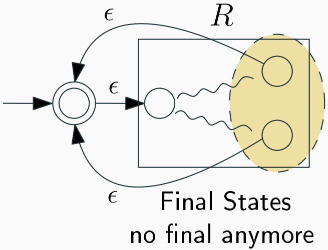


Concatenation
$R_1 \, . \, R_2$



RE ⇒ Finite Automaton

Alternation $R*$



Exercise

Question

- Construct an NFA for the regular expression $(ab)* \mid b*$

Exercise

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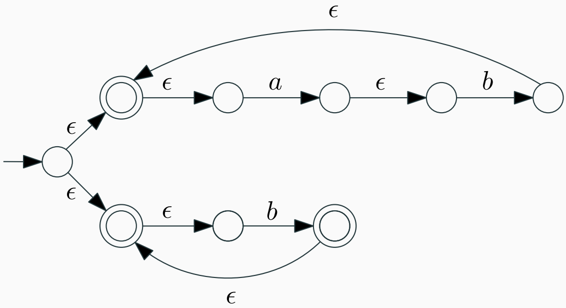
- Construct an NFA for the regular expression $(ab)* \mid b*$

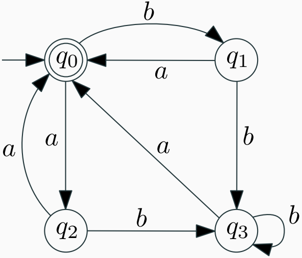
Answer



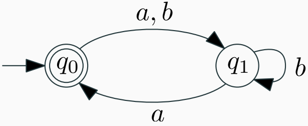
DFA Minimization

- Generated DFAs may have a large number of states
- **DFA Minimization:** Converts a DFA to another DFA that:
  - recognizes the same language
  - has a minimum number of states
- Increases time/space efficiency

DFA 1



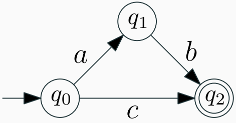
DFA 2

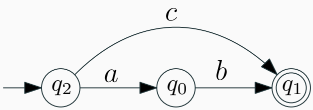


Both DFAs accept: $((a \mid b)b * a)*$

DFA Minimization

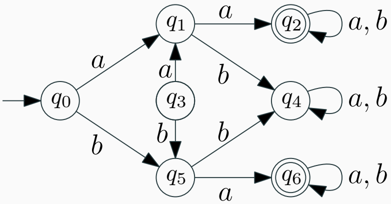
- For every regular language L there exists a unique minimal DFA that recognizes L
  - uniqueness up to renaming of states (isomorphism)
- Minimal DFA can be found mechanically





DFA Minimization: Example

- Remove unreachable states: there is no path from initial state to q_3



$\Sigma = \{a, b\}$

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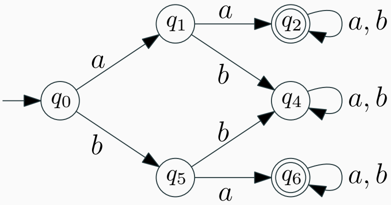
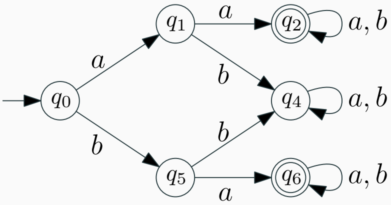


$\Sigma = \{a, b\}$

DFA Minimization: Example

- Remove unreachable states: there is no path from initial state to q_3

- q_2 , q_6 are both accepting sinks with self-loop for any character in Σ
- Any string reaches q_2 or q_6 is guaranteed to be accepted later
- q_2 and q_6 are **equivalent** states: we can unify them



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DFA Minimization: Example

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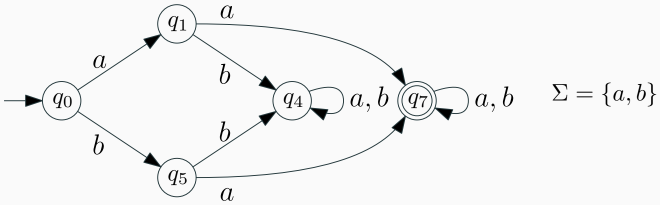

DFA Minimization: Example

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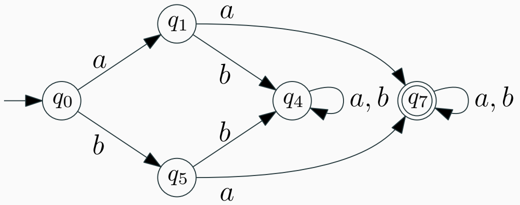
- q_2 and q_6 are **equivalent** states: we can unify them

- If DFA is in q_1 or q_5:
  - if next character is a, it forever accepts in both states
  - if next character is b, it forever rejects in both states

- q_1 and q_5 are **equivalent** states: we can unify them



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DFA Minimization: Example

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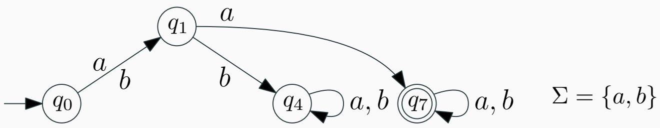


Equivalent States

Intuition

- Two states are equivalent if all subsequent behavior from those states is the same
- Equivalent states may be unified without affecting DFA's behavior

Definition

- We say that states p and q are equivalent if for all w:
  $\hat{\delta}(p, w)$ is an accepting state iff $\hat{\delta}(q, w)$ is an accepting state
- $\hat{\delta}$ is the transition function extended for words

DFA Minimization: Procedure

- Write down all pairs of state as a table
- Every cell in table denotes if corresponding states are equivalent
- Table is initially unmarked
- We mark pair (p_i, p_j) when we discover p_i and p_j are not equivalent

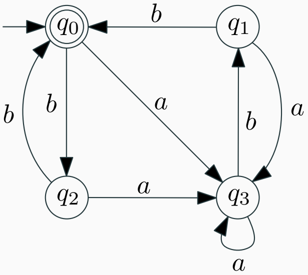
	q_1	q_2	q_3	q_4	q_5	q_6
q_0	?	?	?	?	?	?
q_1		?	?	?	?	?
q_2			?	?	?	?
q_3				?	?	?
q_4					?	?
q_5						?

DFA Minimization: Procedure

1. Start by marking all cells (q_i, q_j) where one of them is final and other is non-final.
2. Look for unmarked pairs (q_i, q_j) such that for some $c \in \Sigma$, the pair $(\delta(q_i, c), \delta(q_j, c))$ is marked. Then mark (q_i, q_j).
3. Repeat step 2 until no such unmarked pairs remain.

Illustration of minimization algorithm

First mark accepting/non-accepting pairs



	q_0	q_1	q_2	q_3
q_0		✓	✓	✓
q_1				
q_2				

Illustration of minimization algorithm

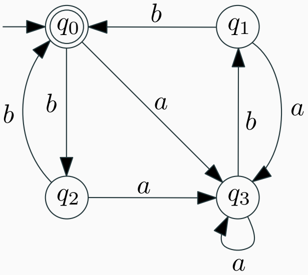
(q_1, q_3) is unmarked,

$q_1 \xrightarrow{b} q_0$,

$q_3 \xrightarrow{b} q_1$,

and (q_0, q_1) is marked,

so mark (q_1, q_3)



	q_0	q_1	q_2	q_3
q_0		✓	✓	✓
q_1				
q_2				

Illustration of minimization algorithm

(q_1, q_3) is unmarked,
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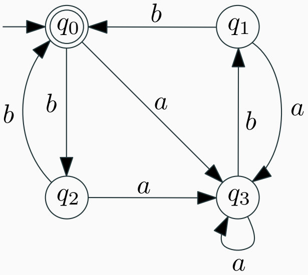
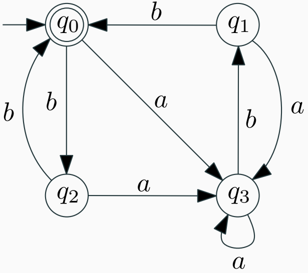
so mark (q_1, q_3)



	q_0	q_1	q_2	q_3
q_0		✓	✓	✓
q_1				✓
q_2				

Illustration of minimization algorithm

(q_2, q_3) is unmarked,
$q_2 \xrightarrow{b} q_0$,
$q_3 \xrightarrow{b} q_1$,
and (q_0, q_1) is marked,
so mark (q_2, q_3)



	q_0	q_1	q_2	q_3
q_0		✓	✓	✓
q_1				✓
q_2				

Illustration of minimization algorithm

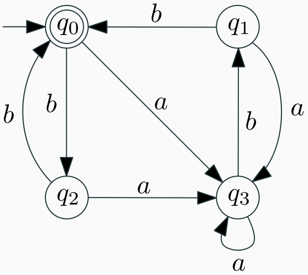
(q_2, q_3) is unmarked,

$q_2 \xrightarrow{b} q_0$,

$q_3 \xrightarrow{b} q_1$,

and (q_0, q_1) is marked,

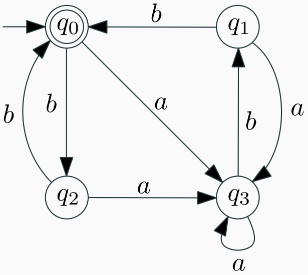
so mark (q_2, q_3)



	q_0	q_1	q_2	q_3
q_0		✓	✓	✓
q_1				✓
q_2				✓

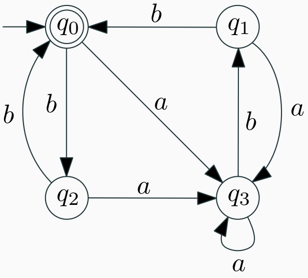
Illustration of minimization algorithm

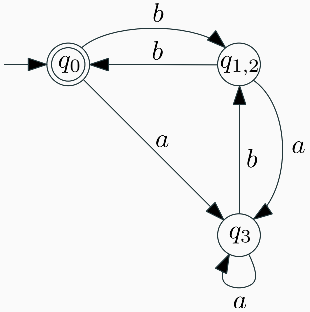
There is no way to mark the only unmarked pair (q_1, q_2)
Obtain minimized DFA by collapsing q_1, q_2 to a single state



	q_0	q_1	q_2	q_3
q_0		$\checkmark$	$\checkmark$	$\checkmark$
q_1				$\checkmark$
q_2				$\checkmark$

Illustration of minimization algorithm





Exercise

Convert the following DFA to a DFA with 3 states