# CSCI 742 - Compiler Construction

Lecture 6

DFA vs. NFA

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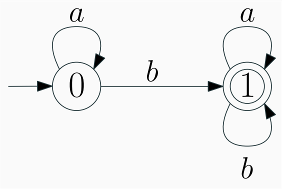
## Recap: Finite State Automaton

$$A = (\Sigma, Q, q_0, \delta, F)$$

- $\Sigma$ — alphabet
- $Q$ — states (nodes in the graph)
- $q_0 \in Q$ — initial state (with $\rightarrow$ sign in drawing)
- $\delta \subseteq Q \times \Sigma \times Q$ — transitions (labeled edges in the graph)
- $F \subseteq Q$ — final states (double circles)




$$\delta = \{(q_0, a, q_0), (q_0, b, q_1),$$
$$(q_1, a, q_1), (q_1, b, q_1)\}$$

# Finite State Automaton (Exercise)

**Question**

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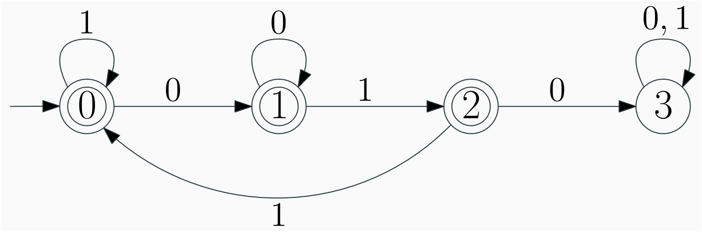
## Answer



# Finite State Automaton (Exercise)

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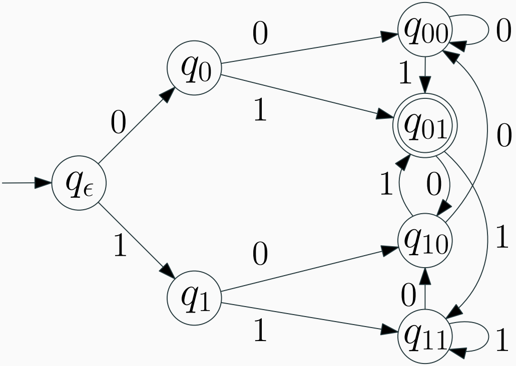


Can you design an automaton for this language with only $3$ states?

# Finite State Automaton (Exercise)

**Question**

- Design an automaton that recognizes all numbers written in binary that are divisible by $2$. For example, the automaton should accept the words $0$, $10$, $100$, $110$, $\cdots$ (leading zeros are ok)

# Finite State Automaton (Exercise)

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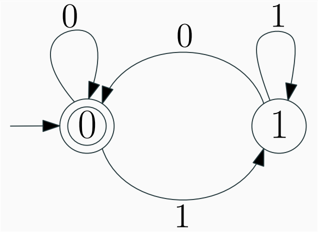
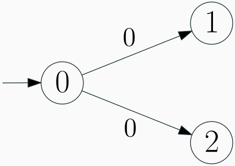


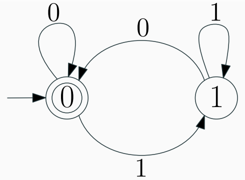
# Types of Finite State Automata

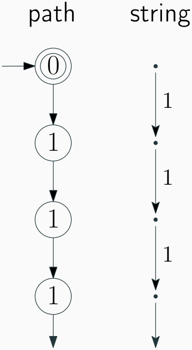
- Deterministic Finite Automata (DFA)
  - $\delta$ is a function $(Q, \Sigma) \mapsto Q$
  - One transition per input per state
  - All examples so far
- Nondeterministic Finite Automata (NFA)
  - $\delta$ is a function $(Q, \Sigma) \mapsto 2^Q$
  - Can have multiple transitions for one input in a given state

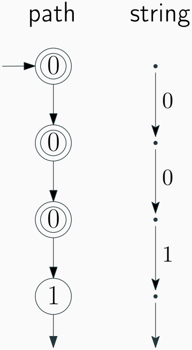


# Computations of a DFA

- For each input string there is exactly one path in a DFA

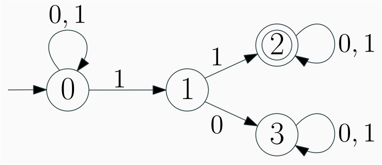


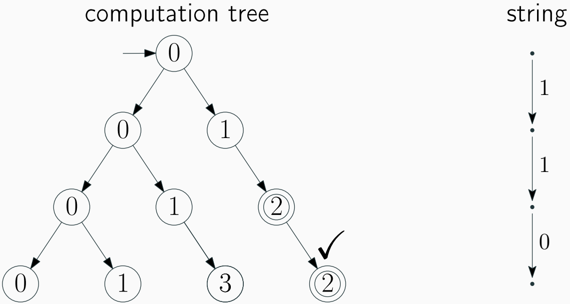



# Computations of an NFA

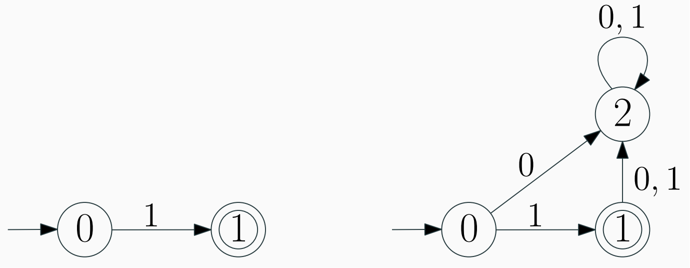
For an input string there are multiple possible computation paths in an NFA




Word is accepted if there is a path in the computation tree that leads to an accepting state

# Undefined Transitions



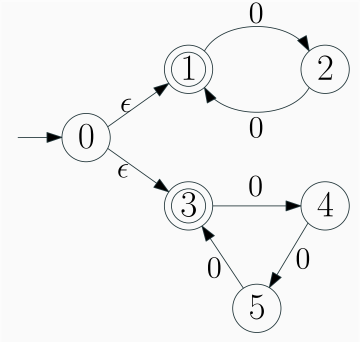

Undefined transitions go to a trap state where no input can be accepted

# $\epsilon$-Transitions

Epsilon transition allows an NFA to change its state spontaneously without consuming any symbol from input

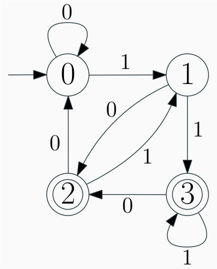
**Example**

NFA that accepts all strings of the form $0^k$ where $k$ is a multiple of $2$ or $3$

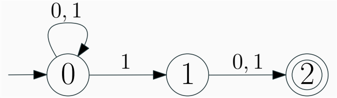


# DFA vs. NFA

**DFA:**



**NFA:**



- NFA for a language can be smaller and easier to construct than DFA
- An implementation of an NFA normally has backtracking
- An implementation of a DFA normally requires only as many steps as the input length

# Exercise

## Question

- Construct an NFA that recognizes all strings over $\Sigma = \{a, b, c\}$ that do not contain all the alphabet symbols $a$, $b$ and $c$.

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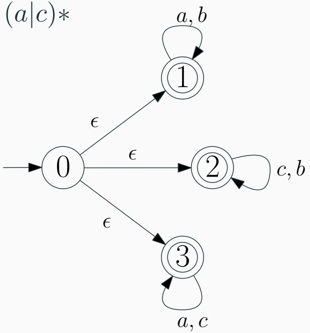
## Answer

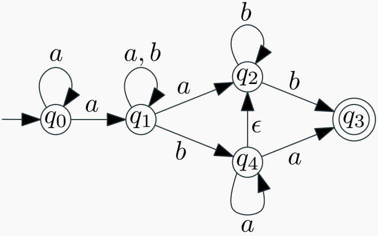
- Let's start with a regular expression
- $(a|b)* \ | \ (b|c)* \ | \ (a|c)*$



# From NFA to DFA

- For every NFA there exists an equivalent DFA that accepts the same set of strings
- NFAs could be exponentially smaller (succinct)
- Idea: keep track of a set of all possible states in which the automaton could be
- View this finite set as one state of new automaton

# From NFA to DFA: Example



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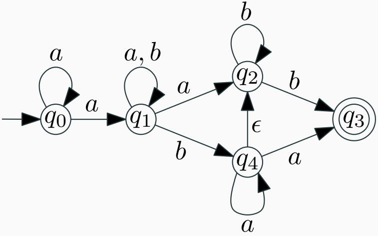
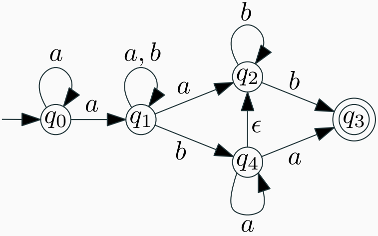



$\rightarrow \{q_0\}$

# From NFA to DFA: Example




$$\begin{tikzcd} \arrow[r] & \{q_0\} \arrow[r, "a"] \arrow[d, "b"] & \{q_0, q_1\} \\ & \times & \end{tikzcd}$$

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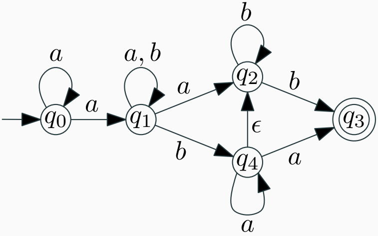


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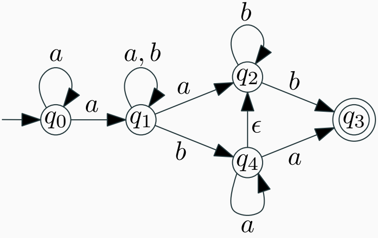


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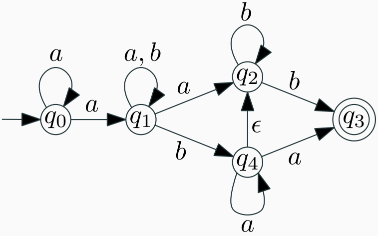


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- When processing if we see a set exactly the same as a set constructed earlier we mark it

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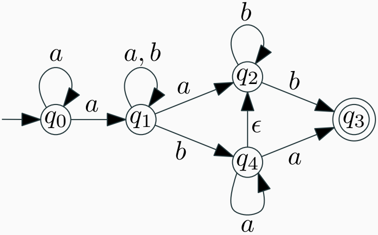


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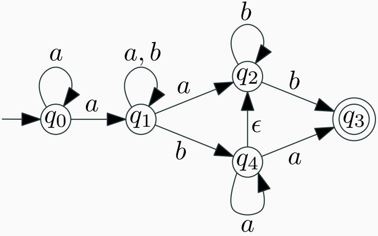


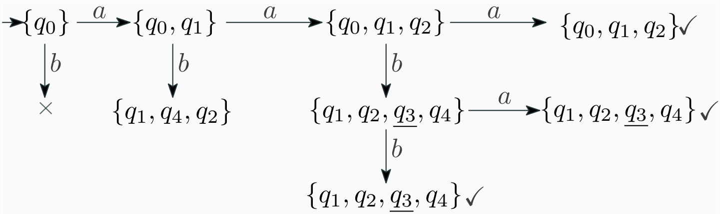


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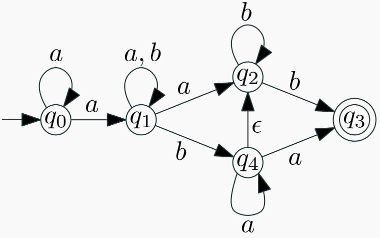


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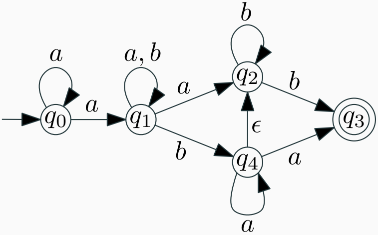


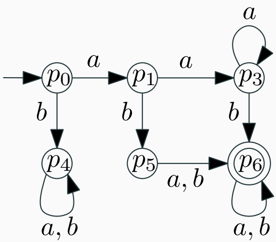


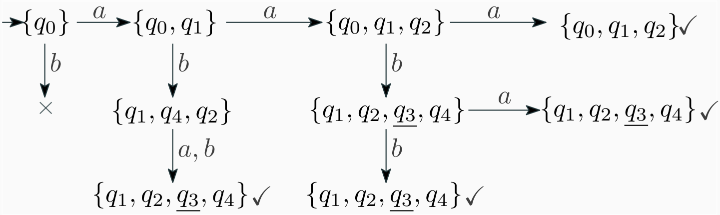


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# Exercise

**Question**

- Construct an NFA for the regular expression $(bb*)|a*$ over alphabet $\{a, b\}$ and determinize it.