CSCI 742 - Compiler Construction

Lecture 35
Data-flow Analysis Framework
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Generalization

Live variable analysis and available expressions analysis are similar

- Define some information that they need to compute
- Build constraints for the information
- Solve constraints iteratively:
  - Information always "increases" during iteration
  - Eventually, it reaches a fixed point

We would like a general framework

- Framework applicable to many other analyses
- Live variable/available expressions instances of the framework

Data-flow Analysis Framework

Data-flow analysis:

- Common framework for many compiler analyses
- Computes some information at each program point
- The computed information characterizes all possible executions of the program

Basic methodology:

- Describe information about the program using an algebraic structure called a **lattice**
- Build constraints that show how instructions and control flow influence the information in terms of values in the lattice
- Iteratively solve constraints

We start by defining lattices and see some of their properties

Partial Orders

A relation $\preccurlyeq \ \subseteq \ D \times D$ on a set D is a **partial order** iff $\preccurlyeq$ is

1. Reflexive: $x \preccurlyeq x$
2. Anti-symmetric: $x \preccurlyeq y$ and $y \preccurlyeq x \Rightarrow x = y$
3. Transitive: $x \preccurlyeq y$ and $y \preccurlyeq z \Rightarrow x \preccurlyeq z$

- A set with a partial order is called a **poset**

Examples:

- If S is a set then $(P(S), \subseteq)$ is a poset
- $(\mathbb{Z}, \leq)$ is a poset

Hasse Diagram

- If $x \preccurlyeq y$ and $x \neq y$, x is predecessor of y
- x immediate predecessor of y: if $x \preccurlyeq y$ and there is no z such that

$$x \preccurlyeq z \preccurlyeq y$$

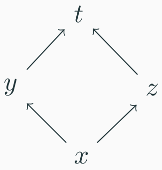
Hasse diagram:

- Directed acyclic graph where the vertices are elements of the set D
- There exists an edge $x \to y$ if x is an immediate predecessor of y

Example.

- $x \preccurlyeq y\ ,\ y \preccurlyeq t\ ,\ z \preccurlyeq t\ ,\ x \preccurlyeq z\ ,\ x \preccurlyeq t$

  $x \preccurlyeq x\ ,\ y \preccurlyeq y\ ,\ z \preccurlyeq z\ ,\ t \preccurlyeq t$



Exercise

- $D_n = \{\text{all divisors of } n\},$ with $d \preccurlyeq d' \Leftrightarrow d \mid d'$
- Draw the Hasse diagram for $D_{12} = \{1, 2, 3, 4, 6, 12\}$

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- $D_n = \{\text{all divisors of } n\},$ with $d \preccurlyeq d' \Leftrightarrow d \mid d'$
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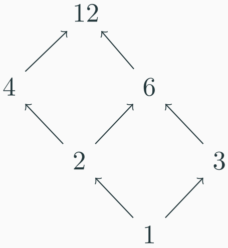


$D_{12} = \{1, 2, 3, 4, 6, 12\}$

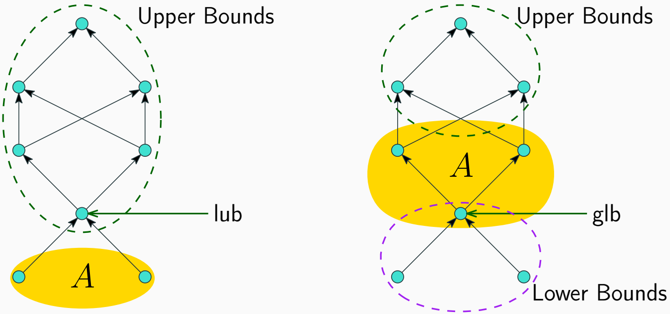
Total Order

- Partial order: no guarantee that all elements can be compared to each other
- Total order (linear order): If for any two elements x and y at least one of $x \preccurlyeq y$ or $y \preccurlyeq x$ is true
- $(\mathbb{N}, \leq)$ is total order
- Hasse diagram is one-track

$$\begin{array}{c} \vdots \\ \uparrow \\ 4 \\ \uparrow \\ 3 \\ \uparrow \\ 2 \\ \uparrow \\ 1 \end{array}$$

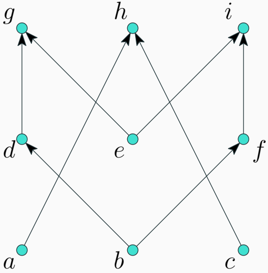
Subset Bounds

- Let $(X, \preccurlyeq)$ be a poset and let $A \subseteq X$ be any subset of X
- An element, $b \in X$, is a **lower bound** of A iff $b \preccurlyeq a$ for all $a \in A$
- An element, $m \in X$, is an **upper bound** of A iff $a \preccurlyeq m$ for all $a \in A$
- An element, $b \in X$, is the **greatest lower bound** (glb) of A iff the set of lower bounds of A is nonempty and if b is the greatest element of this set
- An element, $m \in X$, is the **least upper bound** (lub) of A iff the set of upper bounds of A is nonempty and if m is the least element of this set



Exercise

Find lower/upper bounds and glb/lub for these sets: $\{b, d\}$,$\{a, c\}$,$\{d, e, f\}$



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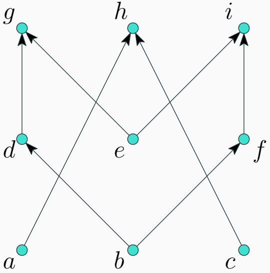

$\{b, d\}$:

- Lower bounds: $\{b\}$ glb: b
- Upper bounds: $\{d, g\}$ lub: d because $d \preccurlyeq g$

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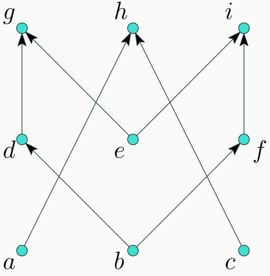
Find lower/upper bounds and glb/lub for these sets: $\{b, d\}$,$\{a, c\}$,$\{d, e, f\}$



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$\{a, c\}$:

- Lower bounds: $\{\}$ no glb
- Upper bounds: $\{h\}$ lub: h

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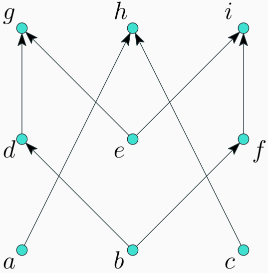

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$\{a, c\}$:

- Lower bounds: $\{\}$ no glb
- Upper bounds: $\{h\}$ lub: h

$\{d, e, f\}$:

- Lower bounds: $\{\}$ no glb
- Upper bounds: $\{\}$ no lub

Lattice

Poset $(D, \preccurlyeq)$ is called a lattice if

- For any $x, y \in D$, $\{x, y\}$ has a lub, which is denoted as $x \sqcup y$ (join)
- For any $x, y \in D$, $\{x, y\}$ has a glb, which is denoted as $x \sqcap y$ (meet)

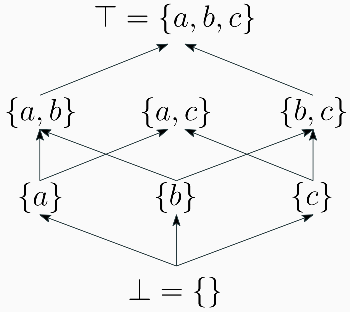
Example.

- For $(P(B), \subseteq)$: $x \sqcap y = x \cap y$, $x \sqcup y = x \cup y$
- For $(\mathbb{Z}, \leq)$: $x \sqcap y = \mathit{min}(x, y)$, $x \sqcup y = \mathit{max}(x, y)$

Complete Lattice

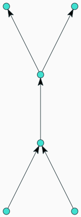
- **Complete lattice** is a poset in which any subset (finite or infinite) has a glb and a lub
  - Every finite lattice is complete
- A complete lattice must have:
  - a least element $\bot$
  - a greatest element $\top$

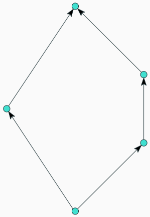
Example: Power Set Lattice

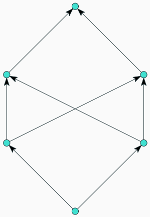


Exercise

- Which are the following posets are lattices?







- To show a poset is not a lattice, it suffices to find a pair that does not have an lub or a glb
- Two elements that don't have an lub or glb cannot be comparable
- View the upper/lower bounds on a pair as a sub-Hasse diagram:
  If there is no greatest/least element in this sub-diagram,
  then it is not a lattice

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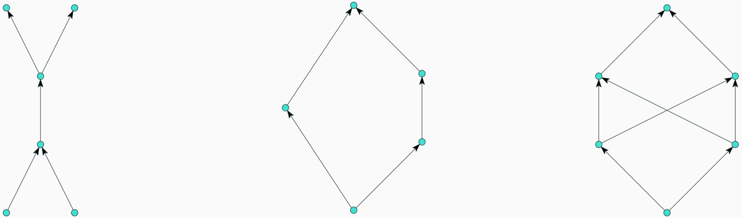

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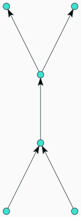

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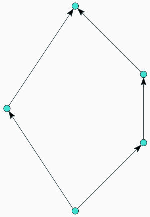
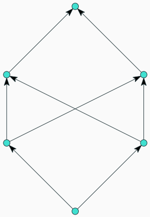


yes ✓



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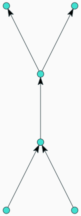

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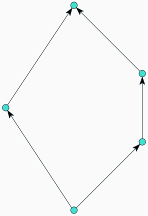

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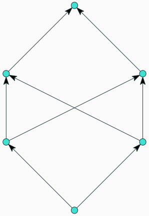

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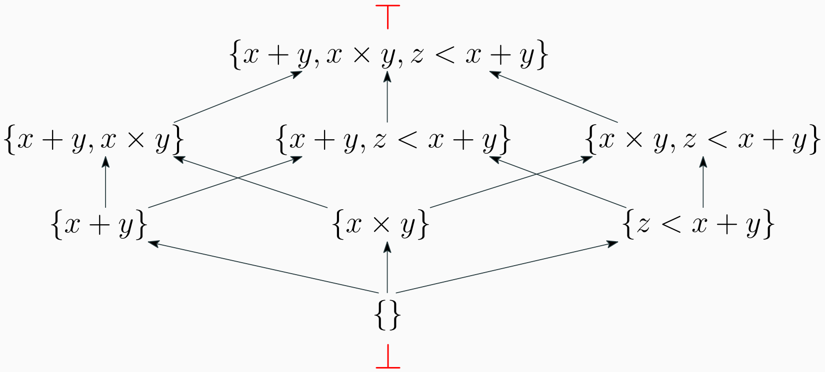
Relation To Data-flow Analysis

- Information computed by e.g. live variable and available expressions analyses can be expressed as elements of lattices
- If $x \leq y$ then x is less or equally precise as y
    - i.e., x is a conservative approximation of y
- Top $\top$: most precise, best case information
- Bottom $\bot$: least precise, worst case information
- Merge function = glb (meet) on lattice elements
    - Most precise element that is a conservative approximation of both elements

Example: Available Expressions



- Trivial answer with zero information, allows no optimization: $\bot = \{\}$ (No expression available)

Example: Live Variables

- If V is the set of all variables in a program and P the power set of V, then $(P, \supseteq)$ is a lattice
- Sets of live variables are elements of this lattice
- Trivial answer with zero information, allows no optimization: $\bot = V$ (All variables are live, nothing is dead)

Using Lattices

- Assume information we want to compute in a program is expressed using a lattice L
- To compute the information at each program point we need to:
  - Determine how each statement in the program changes the information
  - Determine how information changes at join/split points in the control flow

Transfer Functions

- Data-flow analysis defines a transfer function $F : L \to L$ for each statement in the program
- Describes how the statement modifies the information
- Consider *in*(S) as information before S,

  and *out*(S) as information after S
- Forward analysis: *out*$(S) = F($*in*$(S))$
- Backward analysis: *in*$(S) = F($*out*$(S))$

Sequential Composition

- Consider statements $S = S_1;...;S_n$ with transfer functions $F_1,...,F_n$
- *in*(S) is information at the beginning
- *out*(S) is information after at the end

- Forward analysis:

$$\mathit{out}(S) = F_n(\cdots(F_1(\mathit{in}(S)))) = F_n \circ \cdots \circ F_1(\mathit{in}(S))$$

- Backward analysis:

$$\mathit{in}(S) = F_1(\cdots(F_n(\mathit{out}(S)))) = F_1 \circ \cdots \circ F_n(\mathit{out}(S))$$

Split/Join Points

- Data-flow analysis uses meet/join operations at split/join points in the control flow
- Forward analysis:

$$\mathit{in}(S) = \bigsqcap \{\mathit{out}(S') | S' \in \mathit{pred}(S)\}$$

- Backward analysis:

$$\mathit{out}(S) = \bigsqcap \{\mathit{in}(S') | S' \in \mathit{succ}(S)\}$$