Lecture 34
Available Expressions Analysis
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April 18, 2018
Recap: Live Variable Analysis

- Compute which variables are live at each program point
- Live variable information flows backward
- Derive a system of constraints between live variable sets

\[
\begin{align*}
in(S') &= (out(S') \setminus def(S')) \cup use(S') \\
out(S') &= \bigcup_{S_i \in \text{succ}(S)} \text{in}(S_i)
\end{align*}
\]

- Solve constraints in an iterative algorithm
Available Expressions

**Idea:** some computation may be a redundant repetition of earlier computation within the same program

An expression like $x+y$ is available at a statement $S$ if

- Every path from the entry node to $S$ compute $x+y$ before reaching $S$;
- There are no assignments to $x$ or $y$ since the last time $x+y$ was computed on the paths to $S$.

**Optimization:** If an expression is available, don’t need to recompute it.
Example: Available Expression

- Is the expression available?

\[
\begin{align*}
  x &= y + z \\
  w &= v - u \\
  u &= x \times z \\
  t &= x \times z \\
  w &= x - t \\
  y &= x + z \\
  s &= z + u \\
  u &= x \times z + y + z + w
\end{align*}
\]
Example: Available Expression

- Is the expression available?

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\begin{align*}
x &= y + z \\
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t &= x \times z \\
w &= x - t \\
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Example: Available Expression

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s = z + u
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u = x \times z + y + z + w
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Example: Available Expression

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w &= x - t \\
y &= x + z \\
s &= z + u \\
u &= x \times z + y + z + w
\end{align*}
\]
Example: Available Expression

- Is the expression available?

\[ x = y + z \]
\[ w = v - u \]
\[ u = x \times z \]
\[ t = x \times z \]
\[ w = x - t \]
\[ y = x + z \]
\[ s = z + u \]
\[ u = x \times z + y + z + w \]
Example: Available Expression

- Is the expression available?

\[ t = x \times z \]
\[ u = x \times z \]
\[ w = x - t \]
\[ y = x + z \]
\[ s = z + u \]
\[ u = x \times z + y + z + w \]
\[ w = v - u \]
\[ x = y + z \]
Example: Available Expression

• Is the expression available?

\[
\begin{align*}
x &= y + z \\
w &= v - u \\
u &= x \times z \\
t &= x \times z \\
w &= x - t \\
y &= x + z \\
s &= z + u \\
u &= x \times z + y + z + w \\
\end{align*}
\]
Example: Available Expression

- Is the expression available?

\[ x = y + z \]
\[ w = v - u \]
\[ u = x \times z \]
\[ t = x \times z \]
\[ w = x - t \]
\[ y = x + z \]
\[ s = z + u \]
\[ u = x \times z + y + z + w \]
• Is the expression available?

\[
x = y + z \\
w = v - u \\
u = x \times z \\
t = x \times z \\
w = x - t \\
y = x + z \\
s = z + u \\
\]

\[
u = x \times z + y + z + w
\]
• Is the expression available?

\[ x = y + z \]
\[ w = v - u \]
\[ u = x \times z \]
\[ t = x \times z \]
\[ w = x - t \]
\[ y = x + z \]
\[ s = z + u \]
\[ u = x \times z + y + z + w \]

yes
Example: Available Expression

- Is the expression available?
• Is the expression available?

\[ t = x \times z \]
\[ u = x \times z \]
\[ s = z + u \]
\[ y = x + z \]
\[ w = x - t \]
\[ w = v - u \]
\[ x = y + z \]

Yes
Example: Available Expression

- How can we use available expression?

\[
\begin{align*}
x &= y + z \\
w &= v - u \\
u &= x \times z \\
w &= x - t \\
t &= x \times z \\
s &= z + u \\
u &= x \times z + y + z + w \\
y &= x + z
\end{align*}
\]
• How can we use available expression?

\[
\begin{align*}
  x &= y + z \\
  w &= v - u \\
  u &= x \times z \\
  t &= u \\
  w &= x - t \\
  s &= z + u \\
  y &= x + z \\
  u &= x \times z + y + z + w
\end{align*}
\]
How can we use available expression?

- $x = y + z$
- $w = v - u$
- $u = x \times z$
- $t = u$
- $w = x - t$
- $y = x + z$
- $s = z + u$
- $u = x \times z + y + z + w$
Example: Available Expression

- How can we use available expression?

\[
\begin{align*}
x &= y + z \\
w &= v - u \\
u &= x \times z \\
t &= u \\
w &= x - t \\
y &= x + z \\
s &= z + u \\
u &= u + y + z + w
\end{align*}
\]
Available Expression Analysis: Forward

- Available expression analysis is a **forward** data-flow analysis
- Information from past instructions must be propagated forward through the program to discover which expressions are available

```java
int t = x*y;  println(x*y);
if(x*y!=6) ...;
...
int z = x * y;
}
```

- **Unlike** variable liveness, expression availability flows forward through the program
- **Like** liveness, each instruction has an effect on the availability information as it flows past
Available Expression Analysis

- A statement makes an expression **available** when it generates (computes) its current value

```
\begin{align*}
\{\}\quad & \text{generates } x+y \\
\{x+y\}\quad & \text{generates } v+5 \\
\{x+y, v+5\}\quad & \text{generates } z*t \\
\{x+y, v+5, z*t\} \quad & \text{generates } u = v + 5; \text{ and } \text{print}(x+y);
\end{align*}
```
A statement makes an expression **unavailable** when it kills (invalidates) its current value.
Available Expression Analysis

- As in Live Variable Analysis, we create functions $\text{gen}(S)$ and $\text{kill}(S)$ which give the sets of expressions the statement $S$ generates and kills.
- Assignment to a variable $x$ kills all expressions in the program which contain occurrences of $x$ ($E_x$).

$$\text{gen(println(x+5))} = \{x+5\} \quad \text{kill}(x = 3) = \{E_x\}$$

$$\{y-1\} \quad \{y-1, x+5\}$$

$$\{y-1, x+5\} \quad \{y-1\}$$
Available Expression Analysis

- If a statement both generates and kills expressions, remove the killed expressions after adding the generated ones

\[
\begin{align*}
&x = x + y \\
&A_0 \\
&A_2
\end{align*}
\]

\[
\begin{align*}
&\text{compute } (x+y) \\
&A_1 = A_0 \cup \{x+y\} \\
&\text{write}(x) \\
&A_2 = A_1 \setminus E_x
\end{align*}
\]

\[
\begin{align*}
\text{gen}(x = x + y) &= \{x+y\} \\
\text{kill}(x = x + y) &= \{E_x\}
\end{align*}
\]

In general:

- \(\text{in}(S)\): set of available expressions immediately before statement \(S\)
- \(\text{out}(S)\): set of available expressions immediately after statement \(S\)

\[
\text{out}(S) = (\text{in}(S) \cup \text{gen}(S)) \setminus \text{kill}(S)
\]
Multiple Successors

- An expression is available at the beginning of statement $S$ if it is available at the end of all predecessor statements.

\[
in(S) = \bigcap_{S_i \in \text{pred}(S)} \text{out}(S_i)
\]
Data-flow Equations

• Start with CFG and derive a system of constraints between sets of available expressions

\[
\begin{align*}
\text{out}(S) &= (\text{in}(S) \cup \text{gen}(S)) \setminus \text{kill}(S) \\
\text{in}(S) &= \bigcap_{S_i \in \text{pred}(S)} \text{out}(S_i)
\end{align*}
\]

Solve constraints:

• Start with empty set of available expressions at start and universal set of available expressions everywhere else
• Iteratively apply constraints
• Stop when we reach a fixed point
Exercise

Compute the set of available expressions at each point of the program

\[ a = x - y \]

\( A_0 = \{\} \)

\[ [x = 0] \]

\( A_1 = \{\text{all}\} \)

\( A_2 = \{\text{all}\} \)

\[ x = z \]

\( A_4 = \{\text{all}\} \)

\( A_3 = \{\text{all}\} \)

\[ b = x - y \]

\( i = x - y \)

\( A_5 = \{\text{all}\} \)

\[ [i < n] \]

\[ [\neg(i < n)] \]

\( A_7 = \{\text{all}\} \)

\[ c = x - y \]

\( A_9 = \{\text{all}\} \)

\[ i = i + c \]

\( A_{10} = \{\text{all}\} \)

\( d = x - y \)
Exercise

Compute the set of available expressions at each point of the program

\[ a = x - y \]

\[ x \neq 0 \]

\[ x = 0 \]

\[ x = z \]

\[ A_0 = {} \]

\[ A_1 = \{ x - y \} \]

\[ A_2 = \{ \text{all} \} \]

\[ A_3 = \{ \text{all} \} \]

\[ b = x - y \]

\[ i = x - y \]

\[ i < n \]

\[ \neg (i < n) \]

\[ A_4 = \{ \text{all} \} \]

\[ A_5 = \{ \text{all} \} \]

\[ A_6 = \{ \text{all} \} \]

\[ c = x - y \]

\[ d = x - y \]

\[ A_7 = \{ \text{all} \} \]

\[ A_9 = \{ \text{all} \} \]

\[ i = i + c \]

\[ A_{10} = \{ \text{all} \} \]
Exercise

Compute the set of available expressions at each point of the program

```
a = x - y
[x ≠ 0]
A0 = {}

A1 = {x - y}
[x = 0]
ob = x - y
i = x - y
[i < n] [¬(i < n)]
A2 = {x = 0, x - y}
A3 = {all}
A4 = {all}
A5 = {all}
A6 = {all}
A7 = {all}
A8 = {all}
A9 = {all}
A10 = {all}
```
Exercise

Compute the set of available expressions at each point of the program

\[ a = x - y \]
\[ A_0 = \{ \} \]

\[ [x \neq 0] \]
\[ A_1 = \{ x - y \} \]

\[ A_2 = \{ x = 0, x - y \} \]

\[ x = z \]
\[ A_3 = \{ \} \]

\[ b = x - y \]
\[ A_4 = \{ \text{all} \} \]

\[ i = x - y \]
\[ A_5 = \{ \text{all} \} \]

\[ [i < n] \]
\[ A_6 = \{ \text{all} \} \]

\[ c = x - y \]
\[ A_7 = \{ \text{all} \} \]

\[ d = x - y \]
\[ A_8 = \{ \text{all} \} \]

\[ i = i + c \]
\[ A_9 = \{ \text{all} \} \]

\[ A_{10} = \{ \text{all} \} \]
Compute the set of available expressions at each point of the program.

\[
a = x - y
\]

\[
A_0 = \{
\}
\]

\[
A_1 = \{x - y\}
\]

\[
A_2 = \{x = 0, x - y\}
\]

\[
A_3 = \{
\}
\]

\[
A_4 = \{\text{all}\}
\]

\[
A_5 = \{\text{all}\}
\]

\[
A_6 = \{\text{all}\}
\]

\[
A_7 = \{\text{all}\}
\]

\[
A_8 = \{\text{all}\}
\]

\[
A_9 = \{\text{all}\}
\]

\[
A_{10} = \{\text{all}\}
\]
Compute the set of available expressions at each point of the program

\[ a = x - y \]  
\[ [x = 0] \]

\[ A_0 = \{ \} \]
\[ A_1 = \{ x - y \} \]
\[ A_2 = \{ x = 0, x - y \} \]
\[ A_3 = \{ \} \]
\[ A_4 = \{ x - y \} \]
\[ A_5 = \{ \text{all} \} \]
\[ A_6 = \{ \text{all} \} \]
\[ A_7 = \{ \text{all} \} \]
\[ A_8 = \{ \text{all} \} \]
\[ A_9 = \{ \text{all} \} \]
\[ A_{10} = \{ \text{all} \} \]
Exercise

Compute the set of available expressions at each point of the program

\[ a = x - y \]

\[ A_0 = \{ \} \]

\[ [x = 0] \]

\[ A_1 = \{ x - y \} \]

\[ A_2 = \{ x = 0, x - y \} \]

\[ x = z \]

\[ A_3 = \{ \} \]

\[ \ell : [x \neq 0] \]

\[ \text{out}(\ell) = \{ x - y, x \neq 0 \} \]

\[ A_4 = \{ x - y \} \]

\[ \ell' : b = x - y \]

\[ \text{out}(\ell') = \{ x - y \} \]

\[ A_5 = \{ x - y \} \]

\[ i = x - y \]

\[ A_6 = \{ \text{all} \} \]

\[ A_7 = \{ \text{all} \} \]

\[ c = x - y \]

\[ A_8 = \{ \text{all} \} \]

\[ A_9 = \{ \text{all} \} \]

\[ d = x - y \]

\[ A_{10} = \{ \text{all} \} \]

\[ i = i + c \]
Compute the set of available expressions at each point of the program:

- $a = x - y$
- $[x = 0]$
- $A_0 = \{\}$
- $A_1 = \{x - y\}$

- $A_2 = \{x = 0, x - y\}$
- $x = z$
- $\ell : [x \neq 0]$
- $\text{out}(\ell) = \{x - y, x \neq 0\}$

- $A_3 = \{\}$
- $\ell' : b = x - y$
- $\text{out}(\ell') = \{x - y\}$
- $i = x - y$
- $A_4 = \{x - y\}$
- $A_5 = \{x - y\}$

- $[i < n]$
- $A_7 = \{x - y, i < n\}$

- $[\neg(i < n)]$
- $A_6 = \{x - y, \neg(i < n)\}$

- $c = x - y$
- $A_9 = \{x - y, i < n\}$

- $i = i + c$
- $A_{10} = \{x - y\}$
- $A_8 = \{x - y, \neg(i < n)\}$
Exercise

Compute the set of available expressions at each point of the program.

\[
\begin{align*}
A_0 &= \{\} \quad & a &= x - y \\
A_1 &= \{x - y\} \\
A_2 &= \{x = 0, x - y\} \\
A_3 &= \{} \\
A_4 &= \{x - y\} \\
A_5 &= \{x - y\} \\
A_6 &= \{x - y, \neg(i < n)\} \\
A_7 &= \{x - y, i < n\} \\
A_8 &= \{x - y, \neg(i < n)\} \\
A_9 &= \{x - y, i < n\} \\
A_{10} &= \{x - y\}
\end{align*}
\]
Data-flow Equations Comparison

Live Variable Analysis

\[\text{in-live}(S) = (\text{out-live}(S) \setminus \text{def}(S)) \cup \text{use}(S)\]
\[\text{out-live}(S) = \bigcup_{S_i \in \text{succ}(S)} \text{in-live}(S_i)\]

Available Expression Analysis

\[\text{out-avail}(S) = (\text{in-avail}(S) \cup \text{gen}(S)) \setminus \text{kill}(S)\]
\[\text{in-avail}(S) = \bigcap_{S_i \in \text{pred}(S)} \text{out-avail}(S_i)\]