



CSCI 742 - Compiler Construction

Lecture 24

Subtyping

Instructor: Hossein Hojjat

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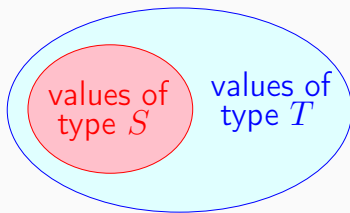
Subtyping

Type: a set of values together with a set of valid operations on those values

Subtype: $S <: T$

A value of type S may be used wherever a value of type T is expected

- $\text{values}(S) \subseteq \text{values}(T)$



- All operators valid on T values are valid on S values

Subtyping Example

Integer <: Object

```
Object o = new Integer(13); // ok
Integer i = new Object(); // type error
```

short <: int

```
void f(int x) {
    x = 10;
}
short s = 0;
f(s); // ok
```

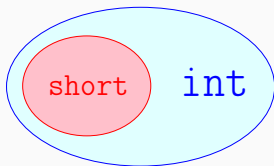
Subtyping Rule

- Rule for subtyping: analogous to set reasoning

$$\frac{\Gamma \vdash e : T_1 \quad T_1 <: T_2}{\Gamma \vdash e : T_2}$$

In terms of sets

$$\frac{\Gamma \vdash e \in T_1 \quad T_1 \subseteq T_2}{\Gamma \vdash e \in T_2}$$



Types for Positive and Negative Ints

$$\text{int} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

$$\text{pos} = \{1, 2, \dots\} \quad (\text{not including zero})$$

$$\text{neg} = \{\dots, -2, -1\} \quad (\text{not including zero})$$

types:

$$\text{pos} <: \text{int}$$

$$\text{neg} <: \text{int}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{pos}}{\Gamma \vdash x + y : \text{pos}}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x \times y : \text{neg}}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{pos}}{\Gamma \vdash x/y : \text{pos}}$$

sets:

$$\text{pos} \subseteq \text{int}$$

$$\text{neg} \subseteq \text{int}$$

$$\frac{\Gamma \vdash x \in \text{pos} \quad \Gamma \vdash y \in \text{pos}}{\Gamma \vdash x + y \in \text{pos}}$$

$$\frac{\Gamma \vdash x \in \text{pos} \quad \Gamma \vdash y \in \text{neg}}{\Gamma \vdash x \times y \in \text{neg}}$$

$$\frac{\Gamma \vdash x \in \text{pos} \quad \Gamma \vdash y \in \text{pos}}{\Gamma \vdash x/y \in \text{pos}}$$

Rules for Positive and Negative Ints

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x + y : ???}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x \times y : ???}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x + y : ???}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x \times y : ???}$$

Rules for Positive and Negative Ints

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$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x + y : ???}$$

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Rules for Positive and Negative Ints

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$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x + y : ???}$$

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Rules for Positive and Negative Ints

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x + y : \text{int}}$$

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$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x + y : \text{int}}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x \times y : ???}$$

Rules for Positive and Negative Ints

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x + y : \text{int}}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x \times y : \text{neg}}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x + y : \text{int}}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{int}}{\Gamma \vdash x \times y : \text{int}}$$

More Rules

$$\frac{\Gamma \vdash x : \text{neg} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x \times y : \text{pos}}$$

$$\frac{\Gamma \vdash x : \text{neg} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x + y : \text{neg}}$$

More rules for division:

$$\frac{\Gamma \vdash x : \text{neg} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x/y : \text{pos}}$$

$$\frac{\Gamma \vdash x : \text{pos} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x/y : \text{neg}}$$

$$\frac{\Gamma \vdash x : \text{int} \quad \Gamma \vdash y : \text{neg}}{\Gamma \vdash x/y : \text{int}}$$

- Let x be a variable

$$\frac{\Gamma \vdash x : \text{int} \quad \Gamma \oplus \{(x, \text{pos})\} \vdash e_1 : \text{void} \quad \Gamma \vdash e_2 : \text{void}}{\Gamma \vdash (\text{if } (x > 0) e_1 \text{ else } e_2) : \text{void}}$$

$$\frac{\Gamma \vdash x : \text{int} \quad \Gamma \vdash e_1 : T \quad \Gamma \oplus \{(x, \text{neg})\} \vdash e_2 : \text{void}}{\Gamma \vdash (\text{if } (x \geq 0) e_1 \text{ else } e_2) : \text{void}}$$

Exercise

Using type systems prove there is no division by zero in computing `res`
(initial environment is $\{(x, \text{int}), (y, \text{int}), (res, \text{int})\}$)

```
if (y > 0) {  
  if (x > 0) {  
    res = 10 / (x*y);  
  }  
}
```

Subtyping Example

- Does the last statement type check?

```
pos f(int x) {  
  if (x < 0) return -x; else return x+1;}  
pos p;  
int q;  
q = f(p);
```

Initial environment: $\Gamma = \{(q, \text{int}), (p, \text{pos}), (f, \text{int} \rightarrow \text{pos})\}$

$$\frac{\frac{\frac{(q, \text{int}) \in \Gamma}{\Gamma \vdash q : \text{int}} \quad \frac{\frac{\Gamma \vdash p : \text{pos} \quad \text{pos} <: \text{int}}{\Gamma \vdash p : \text{int}} \quad \Gamma \vdash f : \text{int} \rightarrow \text{pos}}{\Gamma \vdash f(p) : \text{pos}} \quad \text{pos} <: \text{int}}}{\Gamma \vdash f(p) : \text{int}}}{\Gamma \vdash (q = f(p)) : \text{void}}$$

Subtyping Example

- Does the last statement type check?

```
pos f(pos x) {  
    if (x < 0) return -x; else return x+1;}  
int p;  
int q;  
q = f(p);
```

- Rules for checking code must allow a subtype where a supertype was expected
- Old rule for assignment:

$$\frac{x : T \in \Gamma \quad \Gamma \vdash e : T}{\Gamma \vdash x = e}$$

- What do we need to change here?

- Rules for checking code must allow a subtype where a supertype was expected
- New rule for assignment:

$$\frac{\Gamma \vdash e : T' \quad T' <: T \quad (x, T) \in \Gamma}{\Gamma \vdash x = e} = \frac{\Gamma \vdash e : T' \quad T' <: T}{\Gamma \vdash e : T} + \frac{\Gamma \vdash e : T \quad (x, T) \in \Gamma}{\Gamma \vdash x = e}$$

Type Checking in Practice

```
class Assignment extends Statement {
    Symbol x;
    Expression e;
    // ...
    Type typeCheck(Environment gamma) {
        Type tp = e.typeCheck(gamma);
        Type t = x.typeCheck(gamma);
        if (tp.subtype(t))
            return t;
        else
            throw new TypeError("type mismatch in assignment");
    }
}
```

$$\frac{\Gamma \vdash e : T' \quad T' <: T \quad (x, T) \in \Gamma}{\Gamma \vdash x = e}$$

Parametrized Types

- Suppose we know that $S <: T$ (S is a subtype of T)
 - Given a parameterized type constructor $\text{TYCON}[T]$, there are three possibilities for the relationship between $\text{TYCON}[S]$ and $\text{TYCON}[T]$
1. **Invariant:** $\text{TYCON}[S]$ and $\text{TYCON}[T]$ are unrelated
 2. **Covariant:** $\text{TYCON}[S] <: \text{TYCON}[T]$
 3. **Contravariant:** $\text{TYCON}[S] :> \text{TYCON}[T]$

Subtyping for Products

- Type for a tuple

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash (e_1, e_2) : T_1 \times T_2}$$

$$\frac{\frac{\Gamma \vdash e_1 : T_1 \quad T_1 <: T'_1}{\Gamma \vdash e_1 : T'_1} \quad \frac{\Gamma \vdash e_2 : T_2 \quad T_2 <: T'_2}{\Gamma \vdash e_2 : T'_2}}{\Gamma \vdash (e_1, e_2) : T'_1 \times T'_2}$$

- Covariant subtyping for Product of types

$$\frac{T_1 <: T'_1 \quad T_2 <: T'_2}{T_1 \times T_2 <: T'_1 \times T'_2}$$

Analogy with Cartesian Product

$$\frac{T_1 <: T'_1 \quad T_2 <: T'_2}{T_1 \times T_2 <: T'_1 \times T'_2}$$

$$\frac{T_1 \subseteq T'_1 \quad T_2 \subseteq T'_2}{T_1 \times T_2 \subseteq T'_1 \times T'_2}$$

