Lecture 21
Introduction to Type Checking
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Compiler Phases

Source Code (concrete syntax)

```java
if (x == 0) x = x + 1;
```

Token Stream

```java
if (x == 0) x = x + 1;
```

Abstract Syntax Tree (AST)

```java
IF
  ==
    x
    0
  ==
    x
    +
      x
      1
```

Attributed AST

```java
IF
  boolean
    ==
      x
      int
    0
      int
    =
      int
      x
      +
        int
        x
        1
int
```

Machine Code

```
16: iload_2
17: ifne 24
20: iload_2
21: iconst_1
22: iadd
23: istore_2
24: ...
```
• Type theory covers a huge range of topics
• Several lectures in the courses
  - Programming Language Concepts (344)
  - Programming Language Theory (740)
• In this course we do not cover the theoretical aspects of type system design
• We are mostly interested in type checking as a major component of the semantic analysis phase
What is a type?

• Type: a set of values and a set of operations on those values
• Example: Integers
• `int x, y;` means:
  - `x, y ∈ [−2^{31}, 2^{31})`
  - Operations `+ − < <= mod ...` are possible on `x` and `y`
• Type errors:
  improper, type-inconsistent operations during program execution
• Type safety: absence of type errors at run time
How to Ensure Type-Safety?

Bind (assign) types, then check types

Type binding

- Defines types for constructs in the program (e.g., variables, functions)
- Can be either explicit (boolean x) or implicit (x = false)
- Type safety: correctness with respect to the type bindings

Type checking

- Static semantic checks to enforce the type safety of the program
- Enforce a set of type-checking rules
Type Check Examples

- Operators (such as +) receive the right types of operands
- User-defined functions receive the right types of operands
- LHS of an assignment should be “assignable”
- Variables are assigned the expected kinds of values
- Return statement must agree with return type
- Class members accessed appropriately
Static vs. Dynamic Typing

- **Statically** typed language: types are defined and checked at compile-time, and do not change during the execution of the program
  - E.g., C, Java, Pascal
- **Dynamically** typed language: types defined and checked at run-time, during program execution
  - E.g., Lisp, Scheme, Smalltalk
Why Static Checking?

- Efficient code: dynamic checks slow down the program
- Guarantees that all executions will be safe
- With dynamic checking, you never know when the next execution of the program will fail due to a type error

Drawbacks

- Adds an annotation burden for programmers
- Static type safety is a conservative approximation of the values that may occur during all possible executions
- It may reject some type-safe programs unfairly
Suitable Formalism

- We have used the following formal notations for specifying the first two phases of compiler:
  - Regular expressions for lexical analysis
  - Context-free grammars for parsers
- We use **inference systems** from logic to formalize type checking
  - Similar to what we did in name analysis
- Inference systems are suitable for performing computations of form:

  If the first expression is of type $T$ and the second expression is of type $T'$ then the third expression must be of type $T''$
Background: Inference Systems

- Example inference rule:

  All great universities have smart students \hspace{1cm} \text{Premise 1}
  RIT is a great university \hspace{1cm} \text{Premise 2}
  RIT has smart students \hspace{1cm} \text{Conclusion}

- Example inference rule:

  \(e_1\) has type \(\text{int}\) \hspace{1cm} \text{Premise 1}
  \(e_2\) has type \(\text{int}\) \hspace{1cm} \text{Premise 2}
  \(e_1 + e_2\) has type \(\text{int}\) \hspace{1cm} \text{Conclusion}
An inference system has two parts:

1. Definition of **Judgments**
   - Judgment: statement asserting a certain fact for an object

2. Finite set of **Inference Rules**

An inference rule has:

1. a finite number of judgments $P_1, P_2, \cdots, P_n$ as premises;
2. a single judgment $C$ as conclusion

If a rule has no premises, it is called an **axiom**

\[
\begin{array}{cccc}
P_1 & P_2 & \cdots & P_n \\
\hline
\multicolumn{4}{c}{(Rule name)} \\
\hline
C
\end{array}
\]
Example: Use an inference system to define the set of even numbers

- Judgment: $\text{Even}(n)$ asserts that $n$ is an even number
- Inference rules:
  - Axiom: 
    \[
    \frac{\text{Even}(0)}{\text{Even}(0) \quad \text{(Even0)}}
    \]
  - Successor Rule: 
    \[
    \frac{\text{Even}(n)}{\text{Even}(n + 2) \quad \text{(EvenS)}}
    \]
• To derive more judgments we create **trees** of inference rules

\[
\begin{align*}
&\text{Even}(0) \quad \text{(Even0)} \\
&\text{Even}(n) \quad \frac{\text{Even}(n + 2)}{\text{Even}(n + 2)} \quad \text{(EvenS)}
\end{align*}
\]
To derive more judgments we create trees of inference rules:

- $\text{Even}(0)$ (Even0)
- $\frac{\text{Even}(n)}{\text{Even}(n + 2)}$ (EvenS)

- Does $\text{Even}(1)$ hold?
- No, because there exists no possible derivation
Derivation Tree

Axioms

Judgment

Judgment

Rules

Judgment

Judgment

Judgment

Judgment

Judgment
Example: Use an inference system to define the less-than relation

- Judgment: \( n < m \) asserts that \( n \) is smaller than \( m \)
- Inference rules:
  - Axiom: 
    \[
    n < n + 1 \quad \text{(Suc)}
    \]
  - Transitivity Rule:
    \[
    \frac{k < n \quad n < m}{k < m} \quad \text{(Trans)}
    \]

Exercise: Prove \( 0 < 3 \).
Type Judgments and Type Rules

- $e$ type checks to $T$ under $\Gamma$ (type environment)

\[ \Gamma \vdash e : T \]

- Types of constants are predefined
- Type binding: types of variables are specified in the source code

- If $e$ is composed of sub-expressions

\[ \frac{\Gamma \vdash e_1 : T_1 \quad \cdots \quad \Gamma \vdash e_n : T_n}{\Gamma \vdash e : T} \]
Type Judgments and Type Rules

\[ \Gamma \vdash e : T \]

If the (free) variables of \( e \) have types given by the type environment gamma, then \( e \) (correctly) type checks and has type \( T \)

\[ \begin{align*}
\Gamma \vdash e_1 : T_1 \\
\vdots \\
\Gamma \vdash e_n : T_n
\end{align*} \]

\[ \Gamma \vdash e : T \]

If \( e_1 \) type checks in \( \Gamma \) and has type \( T_1 \)
and ...
and \( e_n \) type checks in \( \Gamma \) and has type \( T_n \)
then \( e \) type checks in \( \Gamma \) and has type \( T \)
Type Rules with Environment

int x;
int y;

Type Environment $\Gamma$

$(x < y) \ ? \ x \ : \ (y + 1)$

Type Rules:

$\frac{(x : T) \in \Gamma}{\Gamma \vdash x : T}$

$\frac{\text{IntConst}(k) : \text{int}}{}$

$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash (e_1 < e_2) : \text{boolean}}$

$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash (e_1 + e_2) : \text{int}}$

$\frac{\Gamma \vdash b : \text{boolean} \quad \Gamma \vdash e_1 : T \quad \Gamma \vdash e_2 : T}{\Gamma \vdash (b \ ? \ e_1 \ : \ e_2) : T}$