CSCI 742 - Compiler Construction

Lecture 9
Context-Free Grammars
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Compiler Phases

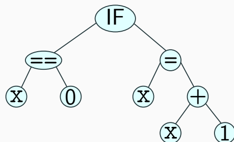
Source Code
(concrete syntax)

`if (x==0) x=x+1;`

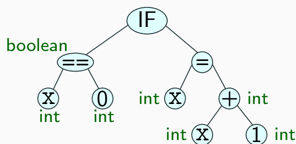
Token Stream

`if` `(` `x` `==` `0` `)` `x` `=` `x` `+` `1` `;`

Abstract Syntex Tree
(AST)

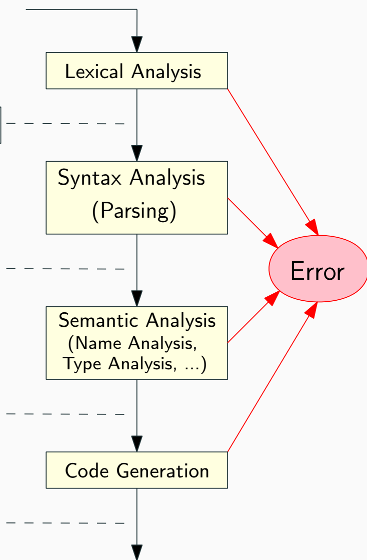


Attributed AST



Machine Code

```
16: iload_2
17: ifne   24
20: iload_2
21: iconst_1
22: iadd
23: istore_2
24: ...
```



Syntax Analysis Example

Source Code
(token stream)

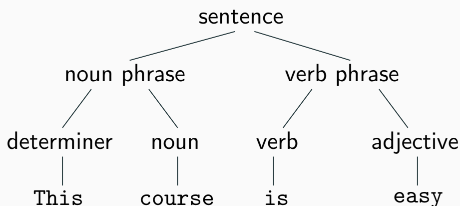
```
{
  if(b) x = x + y;
  while (x > 5) {
    System.out.println(x);
    x = x - 1;
  }
}
```

Abstract
Syntax Tree
(AST)

Abstract syntax tree removes extra syntax (e.g. parenthesis)

Syntax Analysis Analogy

- Syntax analogy for natural languages recognizes whether a sentence is grammatically well-formed



Syntax Analysis Scope

- Parsing only checks syntax correctness
- Several important inspections are deferred until later phases
  - e.g. semantic analysis is responsible for type checking

Program with correct syntax:

```
int x = true;      // type not agree
int y;             // variable not initialized
x = (y < z);       // variable not declared
```

Overview of Syntactic Analysis

- **Input:** Stream of tokens
- **Output:** Abstract Syntax Tree (AST)

What we need for syntax analysis:

- Expressive description technique: describe the syntax
- Acceptor mechanism: determine if input token stream satisfies the syntax description

For lexical analysis:

- Regular expressions describe tokens
- Finite Automata is acceptor for regular expressions

Specifying Language Syntax

- First problem:
  how to describe language syntax precisely and conveniently
- Regular expressions can describe tokens expressively
- Regular expressions are
  - easy to implement
  - efficient by converting to DFA
- Why not use regular expressions (on tokens) to specify programming language syntax?

Limits of REs

- Programming languages are not regular:
  cannot be described by regular expressions
- Consider nested constructs (blocks, expressions, statements)
- Example: language of balanced parentheses is not regular
  () (()) ()()() (())()((()()))
  (( )(()) (()())
- Problem: acceptor needs to keep track of number of parentheses seen so far: unbounded counting
- Automaton has finite memory, cannot count

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  (( )(())   (()())
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- **Question:** How can we show that a language is non-regular?
- **Answer:** Pumping Lemma
  (refer to Computer Science Theory course)

Context-free Grammars

- We use context-free grammars instead of finite state automata
- A specification of the balanced-parenthesis language using context-free grammar

$$S \to (S)$$
$$S \to SS$$
$$S \to \epsilon$$

- If a grammar accepts a string, there is a derivation of that string using the rules of the grammar

$$S \Rightarrow SS \Rightarrow (S)S \Rightarrow ((S))S \Rightarrow (())S \Rightarrow (())(S) \Rightarrow (())()$$

Context-free Grammars

A context-free grammar is a 4-tuple $G = (T, N, S, R)$ where

- T: token or ϵ
- N: Non-terminal symbols: syntactic variables
- S: Start symbol: special non-terminal
- R: Production rule of the form LHS $\rightarrow$ RHS
  - LHS: single non-terminal
  - RHS: a string of terminals and non-terminals

Context-free Grammars: Remark

- Vertical bar is shorthand for multiple production rules
- We abbreviate

$$S \to p$$
$$S \to q$$

- as $\quad S \to p \mid q$

Context-free Grammars

- Production rule specifies how non-terminals can be expanded
- A derivation in G starts from the starting symbol S
- Each step replaces a non-terminal with one of its right hand sides
- Language $L(G)$ of a grammar G:
  set of all strings of terminals derived from the start symbol

Exercise

Give context-free grammars that generate the following languages under $\Sigma = \{0, 1\}$

1. all strings that contain at least three 1s

2. all strings with odd length and the middle symbol 0

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$$S \to X1X1X1X$$
$$X \to 0X \mid 1X \mid \epsilon$$

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1. all strings that contain at least three 1s

$$S \to X1X1X1X$$
$$X \to 0X \mid 1X \mid \epsilon$$

2. all strings with odd length and the middle symbol 0

$$S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$$

RE is a subset of CFG

- Inductively build a production rule for each regular expression operator

ϵ	$S \rightarrow \epsilon$
$\texttt{a}$	$S \rightarrow \texttt{a}$
$R_1 R_2$	$S \rightarrow S_1 S_2$
$R_1 \vert R_2$	$S \rightarrow S_1 \vert S_2$
$R_1 *$	$S \rightarrow S_1 S \vert \epsilon$

where

- G_1: grammar for R_1, with start symbol S_1
- G_2: grammar for R_2, with start symbol S_2

Derivation Example

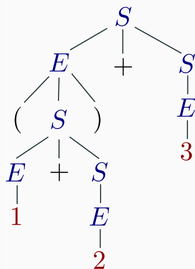
- Grammar:

$$S \rightarrow E + S \mid E$$
$$E \rightarrow \textsf{number} \mid (S)$$

- Derive: $(1 + 2) + 3$

$$S \Rightarrow E + S \Rightarrow (S) + S \Rightarrow (E + S) + S$$
$$\Rightarrow (1 + S) + S \Rightarrow (1 + E) + S \Rightarrow (1 + 2) + S$$
$$\Rightarrow (1 + 2) + E \Rightarrow (1 + 2) + 3$$

Derivation ⇒ Parse Tree



- Parse Tree: tree representation of derivation
- Leaves of tree are terminals
- Internal nodes: non-terminals
- No information on order of derivation steps

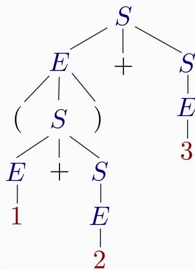
Derivation

$$S \Rightarrow E + S \Rightarrow (S) + S \Rightarrow (E + S) + S$$
$$\Rightarrow (1 + S) + S \Rightarrow (1 + E) + S \Rightarrow (1 + 2) + S$$
$$\Rightarrow (1 + 2) + E \Rightarrow (1 + 2) + 3$$

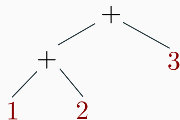
Another Derivation

$$S \Rightarrow E + S \Rightarrow (S) + S \Rightarrow (E + S) + S$$
$$\Rightarrow (E + E) + S \Rightarrow (E + E) + E \Rightarrow (1 + E) + E$$
$$\Rightarrow (1 + 2) + E \Rightarrow (1 + 2) + 3$$

Parse Tree vs. AST



Parse Tree (Concrete Syntax)



Abstract Syntax Tree

Discards nonessential information