CSCI 742 - Compiler Construction

Lecture 6

DFA vs. NFA

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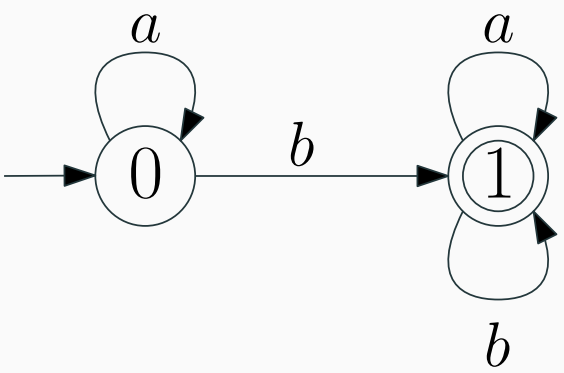
February 3, 2017

Recap: Finite State Automaton

$A = (\Sigma, Q, q_0, \delta, F)$

- Σ alphabet
- Q states (nodes in the graph)
- $q_0 \in Q$ initial state (with $\rightarrow$ sign in drawing)
- $\delta \subseteq Q \times \Sigma \times Q$ transitions (labeled edges in the graph)
- $F \subseteq Q$ final states (double circles)



$$\delta = \{(q_0, a, q_0), (q_0, b, q_1), (q_1, a, q_1), (q_1, b, q_1)\}$$

Finite State Automaton (Exercise)

Question

- Design an automaton that recognizes strings over $\Sigma = \{0, 1\}$ that do not contain the substring 010

Finite State Automaton (Exercise)

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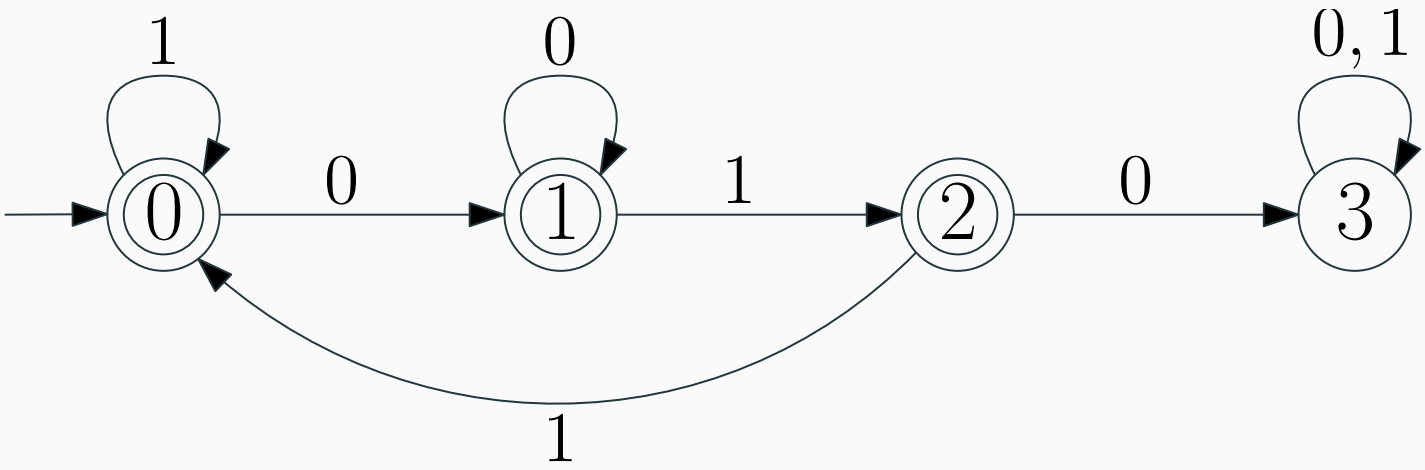
Answer



Finite State Automaton (Exercise)

Question

- Design an automaton that recognizes strings over $\Sigma = \{0, 1\}$ that end in the substring 01

Finite State Automaton (Exercise)

Question

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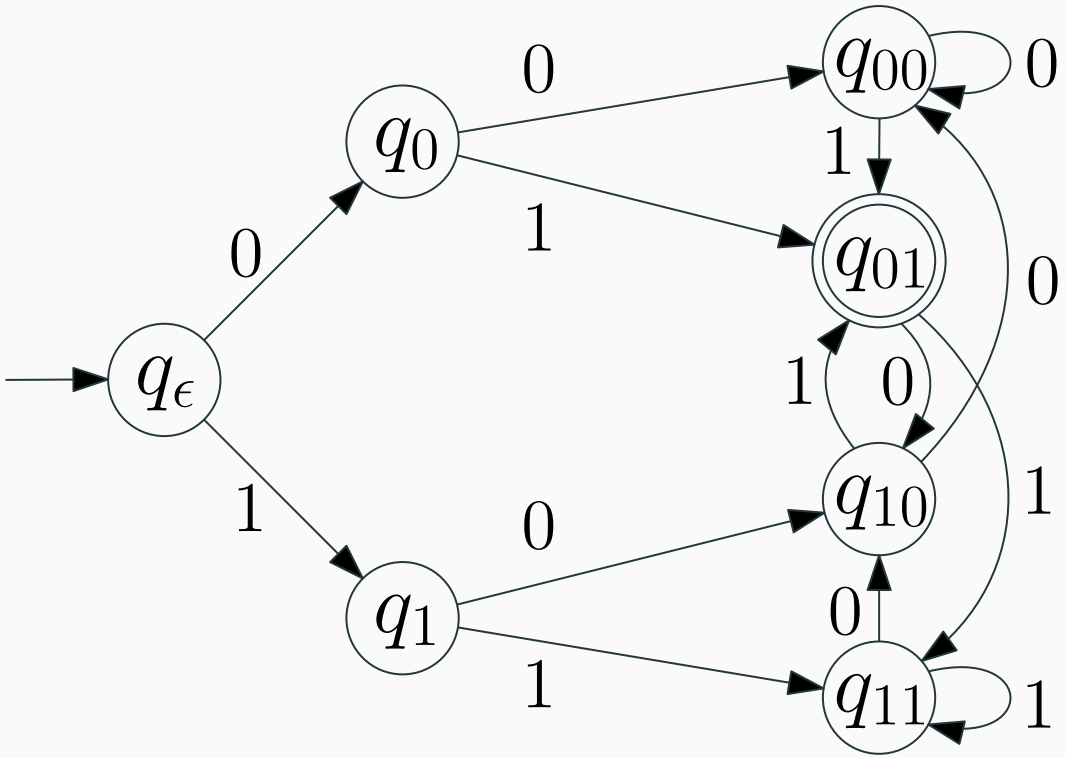
Answer



Finite State Automaton (Exercise)

Question

- Design an automaton that recognizes all numbers written in binary that are divisible by 2.
  For example, the automaton should accept the words 0, 10, 100, 110, $\cdots$
  (leading zeros are ok)

Finite State Automaton (Exercise)

Question

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  For example, the automaton should accept the words 0, 10, 100, 110, $\cdots$
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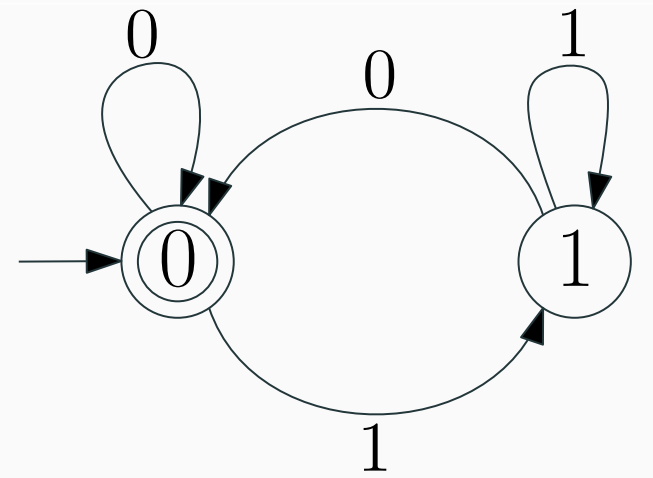
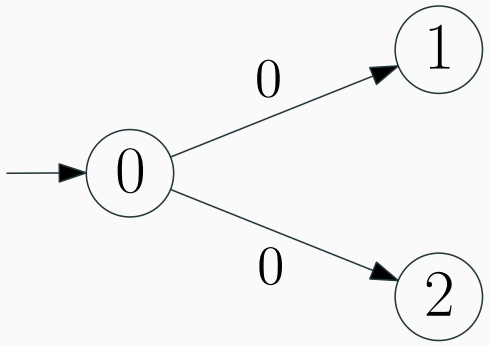
Answer



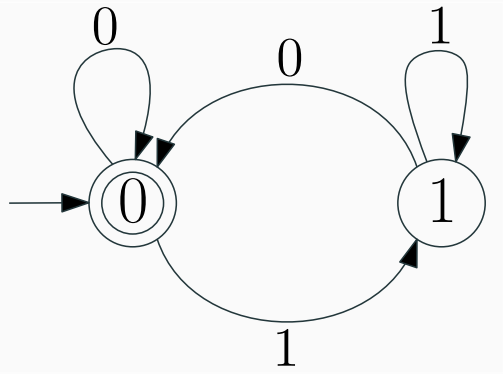
Types of Finite State Automata

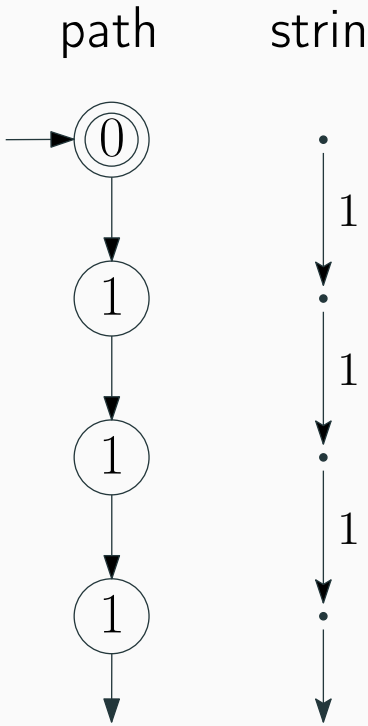
- Deterministic Finite Automata (DFA)
  - δ is a function $(Q, \Sigma) \mapsto Q$
  - One transition per input per state
  - All examples so far
- Nondeterministic Finite Automata (NFA)
  - δ is a function $(Q, \Sigma) \mapsto 2^Q$
  - Can have multiple transitions for one input in a given state

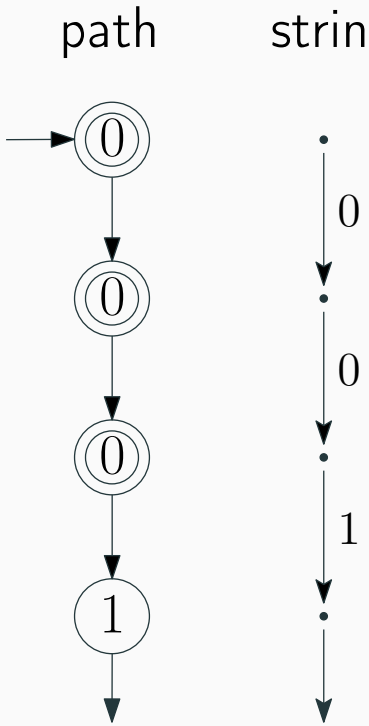


Computations of a DFA

- For each input string there is exactly one path in a DFA

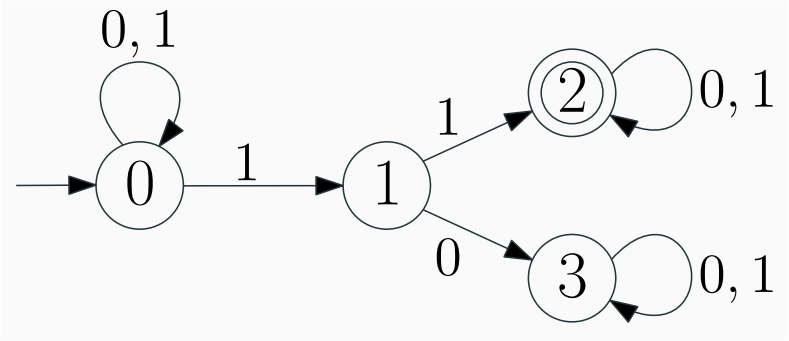




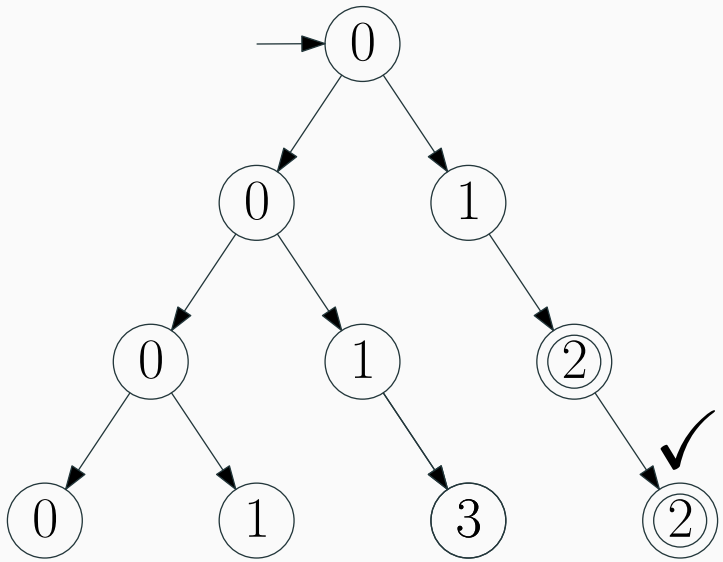


Computations of an NFA

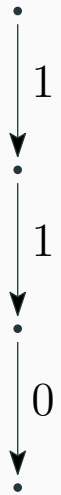
For an input string there are multiple possible computation paths in an NFA



computation tree

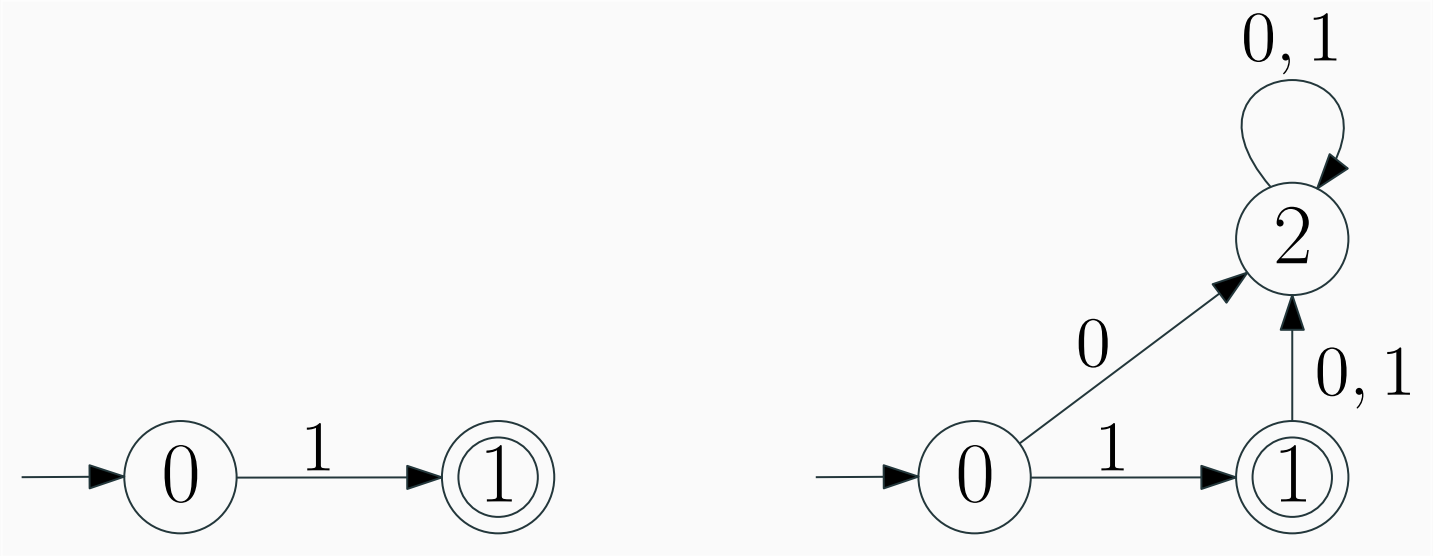


string



Word is accepted if there is a path in the computation tree that leads to an accepting state

Undefined Transitions



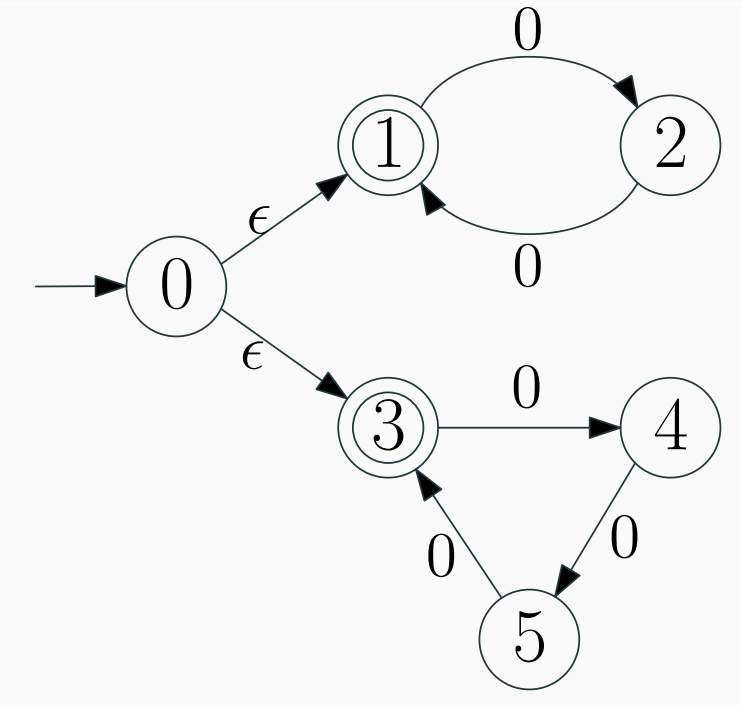
Undefined transitions go to a trap state where no input can be accepted

ϵ-Transitions

Epsilon transition allows an NFA to change its state spontaneously without consuming any symbol from input

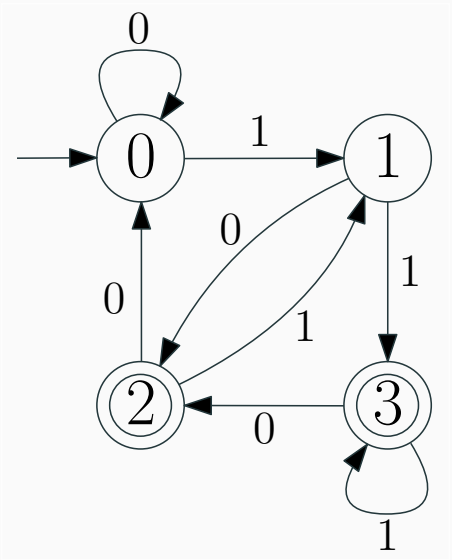
Example

NFA that accepts all strings of the form 0^k where k is a multiple of 2 or 3



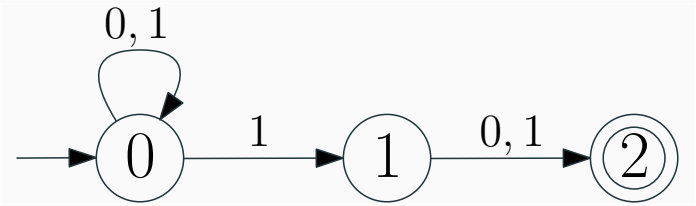
DFA vs. NFA

DFA:



- NFA for a language can be smaller and easier to construct than DFA
- An implementation of an NFA normally has backtracking
- An implementation of a DFA normally requires only as many steps as the input length

NFA:



Exercise

Question

- Construct an NFA that recognizes all strings over $\Sigma = \{a, b, c\}$ that do not contain all the alphabets a, b and c.

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- Construct an NFA that recognizes all strings over $\Sigma = \{a, b, c\}$ that do not contain all the alphabets a, b and c.

Answer

- Let's start with a regular expression
- $(a|b) * \ | \ (b|c) * \ | \ (a|c)*$